1) NMOS: \( \frac{W}{L} = 16.6 \text{ mA/V}^2 \)

Circuit @ \( V_{GS} = V_I + \sqrt{\frac{I_{DS}}{\frac{W}{L}}} (1 + \lambda V_{GS}) \)

\( I_{DS} = 200 \mu A \)

By solving for \( V_{GS} \) iteratively:
(assume an initial value for \( V_{GS} \), e.g. \( V_{GS} = 0 \),
fill it in in the RHS of the equation,
compute the new value of \( V_{GS} \) and repeat)

\( V_{GS} = 0 \rightarrow V_{GS} = 455 \text{ mV} \rightarrow V_{GS} = 449 \text{ mV} \)

\( \rightarrow V_{GS} = 449 \text{ mV} \)

\( R_{REF} = \frac{V_{DD} - V_{GS}}{I_{DS}} = \frac{1.2V - 449 \text{ mV}}{200 \mu A} = 3755 \Omega \)

\( r_o \approx \frac{1}{\lambda I_{DS}} = 25 \text{ k}\Omega \)

\( V_{DSat} = 149 \text{ mV} \)

Saturation range is \([V_{DSat}, V_{DD}] = [149 \text{ mV}, 1.2V]\)

\( (\text{SPICE: [160 mV, 1.2V]} \)

\( R_{out} = r_o \approx 25 \text{ k}\Omega \)

\( (\text{SPICE: 27.2 k}\Omega) \)
circuit ②  \[ V_{SB} = R_S I = 200 \text{mV} \]
\[ V_T = V_{TO} + \sqrt{2qI + V_{SB} - V_{TO}} \]
\[ = 324 \text{mV} \]

\[ V_{GS} - V_T = 149 \text{mV} \quad \text{(identical to circuit ②)} \]
\[ V_{GS} = (V_{GS} - V_T) + V_T = 149 \text{mV} + 324 \text{mV} = 473 \text{mV} \]

\[ R_{REF} = \frac{V_{DD} - V_{GS} - \Delta V_{RS}}{I} \approx \frac{1.2 \text{V} - 473 \text{mV} - 200 \text{mV}}{200 \mu A} \]
\[ R_{REF} = 2635 \Omega \]

\[ r_0 \times \frac{1}{\lambda I_{DS}} = 25 \Omega \]
\[ g_m = \frac{2I_{DS}}{V_{GS} - V_T} = 2.68 \text{mS} \]
\[ I = \sqrt{2qI + V_{SB}} = 0.11 \]

\[ R_{out} = R_S + r_0 + (1 + x)g_m r_0 R_S \]
\[ R_{out} \approx 100 \Omega \quad \text{(SPICE: 1034.2)} \]

\[ \text{saturation range : } [\Delta V_{RS} + V_{DSat}, V_{DD}] = [347 \text{mV}, 1.2 \text{V}] \]
\[ \text{(SPICE: [360 mV, 1.2V])} \]
circuit \[ \text{circuit} \]

\[ V_{G2} = 44.9 \, \text{mV} \quad (\text{cfr. circuit @}) \]

\[ V_{SB2} = V_{G2} = 44.9 \, \text{mV} \]

\[ V_{T4} = V_{10} + \gamma (V_{2} + V_{SB2} - V_{2'}) = 350 \, \text{mV} \]

\[ V_{G5} - V_{T4} = 14.9 \, \text{mV} \]

\[ V_{G5} = (V_{G5} - V_{T4}) + V_{T4} = 149 \, \text{mV} + 350 \, \text{mV} = 449 \, \text{mV} \]

\[ R_{\text{REF}} = \frac{V_{DD} - V_{G5} - V_{G2}}{I} = \frac{1.2 \, \text{V} - 499 \, \text{mV} - 449 \, \text{mV}}{200 \, \text{\mu A}} \]

\[ R_{\text{REF}} = 1.26 \, \text{k}\Omega \]

\[ r_0 = \frac{1}{\lambda I_{D5}} = 25 \, \text{\mu A} \]

\[ g_m = \frac{2I_{DS}}{V_{DSat}} = 268 \, \text{mS} \]

\[ X_4 = \sqrt{\frac{V_{2} + V_{SB2}}{2}} = 0.1 \]

\[ R_{out} = 2 r_0 + (1 + x) g_m r_0^2 \approx (1 + x) g_m r_0^2 \]

\[ R_{out} \approx 1.8 \, \text{M}\Omega \quad (\text{SPICE : } 2 \, \text{M}\Omega) \]

Saturation range: \[ [V_{G5} + V_{D3sat}, V_{DD}] \]

\[ = [648 \, \text{mV}, 1.2 \, \text{V}] \]

(SPICE: [610 \, \text{mV}, 1.2 \, \text{V}])

\[ (V_{D1} = V_{S3} = V_{G3} - V_{G5} = V_{G4} - V_{G5} = V_{S4} + V_{G5} - V_{G3} = V_{S4} = V_{G2} = V_{G3}) \]
The DC problem is identical to that of circuit (a): $R_{REF} = 1.26 \, k\Omega$

(Both problems have 2 stacked diode-connected transistors between $R_{REF}$ and GND: $M_4$ and $M_2$ for problem 1.c, $M_4$ and $M_1$ for problem 1.d)

Because of the same reason, the output range will be the same:

\[ \text{Saturation range: } [V_{ds} + V_{dsat}, V_{dd}] = [648 \, mV, 1.2V] \]

(SPICE: [610 mV, 1.2V])

For the output resistance, we inject a test current into the drain of $M_3$ and calculate all the small signal voltages caused by this current.

The boxed numbers refer to the nodes, and their voltages will be called $V_1, V_2, V_3, V_4$.
- $i_{test}$ will flow through $M_3$ into $N_1$, which is diode connected
  $\Rightarrow V_1 = \frac{i_{test}}{g_m}$

- $V_{gs_2} = V_i \Rightarrow g_m \frac{i_{test}}{g_m} = i_{test}$ will flow out of node 2 into the dependent current source of $M_2$

- Looking down from node 2, we see $R_{o2}$. Looking up from node 2, we see approximately $\frac{1}{g_m} + R_{REF} \ll R_0$

  Therefore, all of the $i_{test}$ current flowing out of node 2, will come from $M_3$ and $R_{REF}$.

  (If you would take the bulk effect into account, you get $\frac{R_{REF} + V_{gs}}{1 + X}$ looking up from node 2)

- $V_3 = -(R_{REF} \cdot i_{test})$
  $V_{gs_3} = -(R_{REF} + \frac{1}{g_m}) \cdot i_{test}$

- $V_{ds_3} = R_0 \left( i_{test} + i_{test} (R_{REF} + \frac{1}{g_m}) g_m \right)$

  original $i_{test} - g_m V_{gs}$

- $\Delta V = V_{ds_3} + V_{ds_1} = \frac{1}{g_m} + 2V_0 + \frac{g_n R_0 R_{REF}}{g_m}$

\[
R_{out} = 2V_0 + \frac{g_m R_0 R_{REF}}{g_m} \approx 1.7 \ \text{k} \Omega \quad (\text{SPICE: } 134 \Omega)
\]
.title homework 5 - problem 1a
.option nomod post
.include ../models.sp
m1 out in 0 0 nmos w=4u l=0.13u
m2 in in 0 0 nmos w=4u l=0.13u
rref vdd in r=3.755k
vdd vdd 0 dc=1.2
vout out 0 dc=0.5
.op
.tf i(m1) vout
.dc vout start=0 stop=1.2 step=0.01
.end

.title homework 5 - problem 1b
.option nomod post
.include ../models.sp
m1 out in s1 0 nmos w=4u l=0.13u
rs1 s1 0 r=1k
rs2 s2 0 r=1k
m2 in in s2 0 nmos w=4u l=0.13u
rref vdd in r=2.635k
vdd vdd 0 dc=1.2
vout out 0 dc=0.7
.op
.tf i(m1) vout
.dc vout start=0 stop=1.2 step=0.01
.end

.title homework 5 - problem 1c
.option nomod post
.include ../models.sp
m1 out1 in1 0 0 nmos w=4u l=0.13u
m2 in1 in1 0 0 nmos w=4u l=0.13u
m3 out in out1 0 nmos w=4u l=0.13u
m4 in in in1 0 nmos w=4u l=0.13u
rref vdd in r=1.26k
vdd vdd 0 dc=1.2
vout out 0 dc=1
.op
.tf i(m3) vout
.dc vout start=0 stop=1.2 step=0.01
.end

.title homework 5 - problem 1d
.option nomod post
.include ../models.sp
m1 out1 out1 0 0 nmos w=4u l=0.13u
m2 in1 out1 0 0 nmos w=4u l=0.13u
m3 out in out1 0 nmos w=4u l=0.13u
m4 in in in1 0 nmos w=4u l=0.13u
rref vdd in r=1.26k
vdd vdd 0 dc=1.2
vout out 0 dc=1
.op
.tf i(m3) vout
.dc vout start=0 stop=1.2 step=0.01
.end
homework 5 - problem 1a

**** small-signal transfer characteristics
i(m1)/vout = 36.7195u
input resistance at vout = 27.2335k
output resistance at i(m1) = 27.2335k

homework 5 - problem 1b

**** small-signal transfer characteristics
i(m1)/vout = 9.0106u
input resistance at vout = 110.9810k
output resistance at i(m1) = 110.9810k

homework 5 - problem 1c

**** small-signal transfer characteristics
i(m3)/vout = 435.4626n
input resistance at vout = 2.2964x
output resistance at i(m3) = 2.2964x

homework 5 - problem 1d

**** small-signal transfer characteristics
i(m3)/vout = 6.8154u
input resistance at vout = 146.7259k
output resistance at i(m3) = 146.7259k
homework 5 - problem 1

Voltage X (lin) (VOLTS)

Result (log)

Voltage=6.10e-01
Resistance=2.06e+06

Voltage=6.10e-01
Resistance=1.34e+05

Voltage=6.10e-01
Resistance=1.03e+05

Voltage=3.60e-01
Resistance=2.72e+04

Voltage=1.60e-01
Resistance=1.03e+05

Voltage=1.60e-01
Resistance=2.72e+04

Circuit a
Circuit b
Circuit c
Circuit d
2)

\[ V_{DD} \]

\begin{align*}
R_D & \\
V_o & \\
V_i & \\
M_1 & \\
M_2 & \\
M_3 & \\
GND & \\
I_B & \\
I_{B3} & \text{=} 400 \mu A
\end{align*}

\[ V_{DD} = 1.2 V \]
\[ R_D = 1 k\Omega \]

Q) Assuming \( M_3 \) is in saturation, the circuit can be simplified as follows:

\[ I_{D3,1,2} = 200 \mu A \]

\[ V_{DSat,1,2} = \sqrt{\frac{2 I_{D3,1,2}}{k' W/L}} = 155 \text{ mV} \]

\[ g_{m,1,2} = \frac{2 I_{DS}}{V_{DSat}} = 2.58 \text{ mS} \]

\[ R_{o,1,2} = \frac{1}{\lambda I_{DS,1,2}} = 25 \text{ k\Omega} \quad (I_{DS,1,2} = 200 \mu A) \]

\[ R_{o3} = \frac{1}{\lambda I_{DS,3}} = 12.5 \text{ k\Omega} \quad (I_{DS,3} = 400 \mu A) \]
differential-mode half-circuit:

\[
\begin{align*}
\text{R}_{\text{out}, \text{dm}} &= 2R_D \parallel 2R_{o,2} = 1.92 \, \text{k}\Omega \\
A_{\text{dm}} &= -g_{m,2} \frac{R_D}{R_{o,2}} \\
&= -2.48 \\
&\quad \text{(SPICE: -2.61)}
\end{align*}
\]

(Note: this is a small-signal circuit, where the transistor stands for its small-signal model)

common-mode half-circuit:

\[
\begin{align*}
\text{R}_{\text{out}, \text{cm}} &= R_D \parallel \left[ g_{m,2} R_0 + R_{o,2} + 2R_{o,3} \right] \\
&\approx R_D = 1k \, \Omega \\
&\quad \text{(SPICE: 999.5} \Omega \\nA_{\text{cm}} &= \frac{-g_{m,2} R_D}{1 + g_{m,2} 2R_{o,3}} \\
&\approx -\frac{R_D}{2R_{o,3}} \\
&= 0.04 \\
&\quad \text{(SPICE: 0.035)}
\end{align*}
\]
lower range of common mode input:
point where \( V_{DS_{3}} = V_{DS_{3}} = \sqrt{\frac{2 I_{DS_{3}}}{\eta \cdot W/L}} \)

\[
V_{IC, min} = V_{DSat_{3}} + V_{S_{1,2}} = V_{DSat_{3}} + V_{T_{1,2}} + V_{DSat_{1,2}} = 220 \text{ mV} + 0.3 \text{ V} + 155 \text{ mV} = 675 \text{ mV}
\]

higher range of common mode input:
point where \( V_{DS_{1,2}} = V_{DSat_{1,2}} \)
(when \( M_{1} \) and \( M_{2} \) go out of saturation)

Once \( M_{3} \) is in saturation, the current through each leg is approximately fixed at 200 \( \mu \)A. This means that \( V_{D_{1,2}} \) is fixed at

\[
V_{D_{1,2}} = V_{DD} - I \cdot R_{D} = 1.2 \text{ V} - 200 \mu \text{A} \cdot 1k \Omega = 1 \text{ V}
\]

\( M_{1} \) and \( M_{2} \) will go out of saturation when

\[
V_{DS_{1,2}} \leq V_{DSat_{1,2}} = V_{S_{1,2}} - V_{T} \quad \Rightarrow \quad V_{D_{1,2}} \leq V_{G_{1,2}} - V_{T} \quad \Rightarrow \quad V_{G_{1,2}} \geq V_{D_{1,2}} + V_{T} = 1 \text{ V} + 0.3 \text{ V} = 1.3 \text{ V}
\]

\[
[V_{IC} \in [675 \text{ mV}, 1.3 \text{ V}]]
\]
For $V_{iD} = 0\, \text{V}$ (and assuming $N_3$ is in saturation) there will be $\frac{400\, \mu\text{A}}{2} = 200\, \mu\text{A}$ flowing through each leg of the differential pair.

For $V_{iD} > 0$, one of the diff pair transistors $N_1$ or $N_2$ will have more current than the other one; the difference in current will get larger when $V_{iD}$ gets larger. This can continue until all the $400\, \mu\text{A}$ flows in one of the legs of the diff pair, and the other leg will be shut off.

This happens for

$$V_{iD} = (V_{GS_{\text{max}}} - V_T) - (V_{GS_{\text{min}}} - V_T)$$

$$= \sqrt{\frac{2 \times 400\, \mu\text{A}}{kW/L}} - 0 = 220\, \text{mV}$$

Then

$$V_{i,\text{max}} = 0.9\, \text{V} + \frac{220\, \text{mV}}{2} = 1.01\, \text{V}$$

($V_D$ will be $1.2\, \text{V} - \frac{2}{1.2} \times 400\, \mu\text{A} = 1.2\, \text{V} - 400\, \text{mV} = 0.8\, \text{V}$ and $V_G < V_D + V_T = 1.1\, \text{V}$, so still saturation)

$$V_{i,\text{min}} = 0.9 - \frac{220\, \text{mV}}{2} = 0.79\, \text{V}$$

Saturation range: $V_{iD} \in [-220\, \text{mV}, +220\, \text{mV}]$
4. Vod versus Vid - plot

The theoretical shape of this curve is derived on page 220 of the textbook:

$$V_{od} = -R_0 \frac{4'}{2} \frac{W}{L} \sqrt{V_{id} \frac{4'}{W/L} - V_{id}^2}$$

Voc versus Vic - plot

Notice how it is impossible to figure out from this plot when H₁ - H₂ go into the linear region: V₀ will remain approximately constant since there will be still 200μA flowing through each drift pair leg, regardless of whether H₁ & H₂ are in the saturation or linear region.
.title homework 5 - problem 2
.option nomod post
.include ../models.sp

* main circuit
m1 vol vil com com nmos w=4u l=0.13u
m2 vo2 vi2 com com nmos w=4u l=0.13u
m3 com bias 0 0 nmos w=4u l=0.13u
m4 bias bias 0 0 nmos w=4u l=0.13u
rd1 vol vdd r=1k
rd2 vo2 vdd r=1k

* input network
vic vic 0 dc=0.9
vid vid 0 dc=0
ed1 vil vic vid 0 0.5 $ +vid/2 to positive input
ed2 vic vi2 vid 0 0.5 $ -vid/2 to negative input

* output network
eco1 voc_1 voc_2 vol 0 0.5 $ construct (vol+vo2)/2
eco2 voc voc_1 vo2 0 0.5 $ current probe (0V v source)
voc voc_2 0 0
fco1 vol gnd voc -1 $ feed current back to circuit
fco2 vo2 gnd voc -1

* supply and bias
ibias vdd bias dc=400u
vdd vdd 0 dc=1.2

* analysis statements
.op
.probe dc vod=v(vol,vo2) voc=v(voc)
.dc vid start=-1.2 stop=+1.2 step=0.01
.dc vic start=0 stop=1.5 step=0.01
.tf v(vol,vo2) vid
*.tf v(voc) vic
.end

*****

homework 5 - problem 2

**** small-signal transfer characteristics
(differential mode)
v(vol,vo2)/vid = -2.6078
input resistance at vid = 1.000e+20
output resistance at v(vol,vo2) = 1.9311k

**** small-signal transfer characteristics
(common mode)
v(voc)/vic = -35.3444m
input resistance at vic = 1.000e+20
output resistance at v(voc) = 999.5331