1) \( V_{GS1} = V_{GS2} + R I_{OUT} \)

\[ V_{T} + V_{DSat1} = V_{T} + V_{DSat2} + R I_{OUT} \]

\[ \sqrt{\frac{2 I_{IN}}{u'(W/L)_1}} = \sqrt{\frac{2 I_{OUT}}{u'(W/L)_2}} + R I_{OUT} \]

Since \( I_{IN} = I_{OUT} \) (current mirror):

\[ \sqrt{\frac{2 I_{OUT}}{u'(W/L)_1}} = \sqrt{\frac{2 I_{OUT}}{u'(W/L)_2}} + R I_{OUT} \]

One trivial solution is \( I_{OUT} = 0 \). The more interesting solution can be found by dividing out \( \sqrt{\frac{2 I_{OUT}}{u'}} \):

\[ \frac{1}{\sqrt{V(W/L)_1}} = \frac{1}{V(W/L)_2} + \sqrt{\frac{k'}{2}} R \sqrt{I_{OUT}} \]

\[ \Rightarrow I_{OUT} = \frac{2}{k' R^2} \left[ \frac{1}{\sqrt{V(W/L)_1}} - \frac{1}{V(W/L)_2} \right]^2 \]
\[ g_{m5} = \sqrt{ \frac{2 \mu' (w/L)_5}{(W/L)_5} I_{DS5} } \]

\[ \frac{I_{DS5}}{(W/L)_5} = \frac{I_{DS1}}{(W/L)_1} \]

\[ g_{m5} = \sqrt{2 \mu' \frac{(W/L)_5^2}{(W/L)_1} I_{DS1}} \]

\[ g_{m5} = \frac{2}{R} \frac{(W/L)_5}{(W/L)_1} \left[ 1 - \sqrt{ \frac{(W/L)_1}{(W/L)_2} } \right] \]

This bias circuit is called a 'constant-\(g_m\)' reference, since \(g_m\) does not depend on electrical transistor parameters, such as \(V_T, \mu'\),... (which are not well controlled), but \(g_m\) only depends on \(W/L\)-ratio's (which are well controlled) and \(R\) (which can be made accurate using special techniques).
2) From the device model: \( W_{N\text{HOS}} = 332 \, \mu\text{A/V}^2 \)
\( W_{P\text{HOS}} = 133 \, \mu\text{A/V}^2 \)

\[ I_{D_{4a}} = 9.375 \, \mu\text{A} \]

\[ V_{\text{DSat}_{4a}} = \sqrt{\frac{2 I_{D_{4a}}}{W_{W/L_{0}}}} = 206 \, \text{mV} \]

\[ V_{\text{DSat}_{4b}} = V_{\text{DSat}_{4a}} \]

\[ V_{\text{DSat}_{2a}} = \sqrt{\frac{2 I_{D_{2a}}}{W_{W/L_{0}}}} = 25 \, \text{mV} \]

\[ V_{\text{DSat}_{2b}} = \sqrt{\frac{2 I_{D_{2b}}}{W_{W/L_{0}}}} = 25 \, \text{mV} \]

Since \( V_{G_{2a}} + V_{G_{2b}} = V_{G_{1a}} + V_{G_{1b}} \)

and \( I_{D_{2a}} = I_{D_{2b}} \) and \( I_{D_{1a}} = I_{D_{1b}} \)

we get \( V_{\text{DSat}_{1a}} = V_{\text{DSat}_{1b}} = 25 \, \text{mV} \)
\( V_{\text{DSat}_{1b}} = V_{\text{DSat}_{2b}} = 25 \, \text{mV} \)

\[ I_{D_{1a}} = I_{D_{2b}} \quad \text{and} \quad I_{D_{2a}} = I_{D_{2b}} \]

(The currents in \( I_{D_{1a}} \) and \( I_{D_{2b}} \) will be slightly larger because of higher \( V_{D_{S}} \))
\[ I_{DS_3} = I_{DS_{4b}} \]

\[ V_{DSat_3} = \sqrt{\frac{2 I_{DS_3}}{W L_{N莫斯} \mu N_{SAT}}} = 206 \text{ mV} \]

The conclusion of \( V_{DSat_{1a}} = V_{DSat_{2a}} \)
and \( V_{DSat_{1b}} = V_{DSat_{2b}} \) can also be derived mathematically:

\[ I_{DS_{2a}} = I_{DS_{2b}} \quad \text{and} \quad I_{DS_{1a}} = I_{DS_{1b}} \quad \text{(for} \ V_{OUT} = 0 \text{V)} \]

\[ \frac{I_{DS_{2a}}}{I_{DS_{1a}}} = \frac{I_{DS_{2b}}}{I_{DS_{1b}}} \]

\[ \frac{V_{DSat_{2a}}^2}{V_{DSat_{1a}}^2} = \frac{V_{DSat_{2b}}^2}{V_{DSat_{1b}}^2} \]

If we call this ratio \( x^2 \), we get

\[ V_{DSat_{1a}} = x \cdot V_{DSat_{2a}} \]
\[ V_{DSat_{1b}} = x \cdot V_{DSat_{2b}} \]

so that \( V_{DSat_{2a}} + V_{DSat_{2b}} = V_{DSat_{1a}} + V_{DSat_{1b}} \)

\[ \iff \quad x = 1 \]

\[ \Rightarrow \quad V_{DSat_{1a}} = V_{DSat_{2a}} \quad \text{and} \quad V_{DSat_{1b}} = V_{DSat_{2b}} \]
(5) \[ V_{IN} = V_{GS3} = V_1 + V_{DSAT3} \approx 500\,m\,V \]

\[ V_{IN} \approx 500\,m\,V \]

(6) **Upper part of the swing:**

- M1b will be cut off
- All \( V_{os} \) stay the same, except for \( V_{os,a} \), which will change greatly with the output current

- We always have:
  \[ |V_{DS4}| + V_{Tn} + V_{DSATa} + \frac{R_{LOAD}}{I_{OUT}} = V_{DD} \]

- Maximum output swing:

  \[ |V_{DSATb}| + V_{IN} + V_{DSATa,max} \]

  \[ + \frac{R_{LOAD}}{I_{OUT, max}} = V_{DD} \]

  \[ = V_{OUT, max} \]

  \[ = R_{LOAD} \frac{W_{MIN}}{2} \frac{L}{L_{sat}} V_{DSATa,max}^2 \]

  \[ \Rightarrow 15 \, V^{-1} V_{DSATa}^2 + V_{DSATa} + (200 \, mV + 300 \, mV - 1.2V) = 0 \]
\[
V_{\text{dsat},a,\text{max}} = \frac{-1 + \sqrt{1 + 4.15 \cdot 0.7}}{2 \cdot 1.5} = 185 \text{ mV}
\]

\[
V_{\text{out}^+,\text{max}} = V_{\text{DD}} - |V_{\text{dsat,a}}| - V_{\text{in}} - V_{\text{dsat},a,\text{max}}
\]

\[
V_{\text{out}^+,\text{max}} = 515 \text{ mV}
\]

Similarly, \( |V_{\text{out}^-,\text{max}}| = +515 \text{ mV} \)

\[
\theta \times \frac{\pi}{4} \frac{V_{\text{out}^-}}{V_{\text{DD}}} = \frac{\pi}{4} \cdot \frac{515 \text{ mV}}{1.8 \text{ V}} = 33.7\%
\]

Using this class B approximations means neglecting the (small) bias currents.

\[
\theta = 33.7\%
\]
If we would like to include the effects of bias currents:

\[ P_L = \frac{V_{out}^2}{2R_{LOAD}} \]

\[ P_{suply} = \frac{2}{\pi} \frac{V_{DD}}{R_{LOAD}} \frac{V_{out}}{V_{DD}} + \frac{V_{DD} \cdot 3I_B}{\text{average of sinusoidal currents in output transistors in } M_{4b} \text{ and } M_{3}} \]

\[ \Rightarrow \eta^{-1} = \frac{P_{suply}}{P_{LOAD}} \]

\[ = \left( \frac{\pi}{4} \frac{V_{out}}{V_{DD}} \right)^{-1} + \frac{6V_{DD}I_B R_{LOAD}}{V_{out}} \]

\[ \Rightarrow \eta = 31\% \]