Effect of Feedback on Frequency Response

\[ v_{IN}(\omega) \rightarrow + \Sigma \rightarrow - a(\omega) \rightarrow v_o(\omega) \]

macro block of \( a(\omega) \)

\[ a_o \rightarrow R \rightarrow C \]
Effect of Feedback on Frequency Response (Cont.)

- **RHP (Right Half-Plane)**
  - Impulse:
  - \( \exp(p_1 \cdot t) \)

- **LHP (Left Half-Plane)**
  - Impulse:
  - \( \exp(p_2 \cdot t) \)
Effect of Feedback on Frequency Response (Cont.)

Let \( a(\omega) \) be a single pole response,

\[
a(s) = \frac{a_o}{1 - \frac{s}{p_1}} \iff a(\omega) = \frac{a_o}{1 + j \frac{\omega}{\omega_{p_1}}}
\]

\[
p_1 = -\omega_{p_1}
\]

\[
\frac{v_{OUT}(s)}{v_{IN}(s)} = A(s) = \frac{a(s)}{1 + a(s) \cdot f} = \frac{1}{f} \left( \frac{T(s)}{1 + T(s)} \right)
\]

\[
A(s) = \frac{a_o}{1 - \frac{s}{p_1}} = \frac{a_o}{1 + a_o \cdot f} \left( \frac{1}{1 - \frac{s}{p_1 \cdot (1 + a_o \cdot f)}} \right)
\]
Effect of Feedback on Frequency Response (Cont.)

Let \( T_o = a_o \cdot f \)

\[
A(s) = \frac{a_o}{1 + T_o} \cdot \left( \frac{1}{1 - \frac{s}{p_1 \cdot (1 + T_o)}} \right)
\]

Pole is at \( p_1 \cdot (1 + T_o) \Rightarrow -\omega_p \cdot (1 + T_o) \)

\[
\infty \leftarrow T_o \quad T_o = 0
\]

Root Locus - motion of poles as loop Gain is increased.
Gain reduction by negative feedback reduces Gain by \( \left( \frac{1}{1 + T_o} \right) \) and increases bandwidth by \( (1 + T_o) \).
Effect of Feedback on Frequency Response (Cont.)

Why not let $T_O \rightarrow \infty$? Problems if we have more than one pole.

\[ a_o \] \[ \frac{a_o}{\left(1 - \frac{s}{p_1}\right) \cdot \left(1 - \frac{s}{p_2}\right) \cdot \left(1 - \frac{s}{p_3}\right)} \]

\[ a(\omega) \]

\( 0.1\omega_{p_1} \quad 0.1\omega_{p_2} \quad 0.1\omega_{p_3} \)

\[ \omega_{p_1} \quad \omega_{p_2} \quad \omega_{p_3} \]

\[ -90^0 \quad -180^0 \quad -270^0 \]

At the frequency \((10\omega_{p_2})\) the phase shift is \(180^0\) or negative feedback at DC is now positive feedback.
Effect of Feedback on Frequency Response (Cont.)

Let's look at the motion of a single pole with positive feedback:

\[ a(\omega) = -\frac{|a_o|}{1 - \frac{s}{p_1}} \]

\[ p_1 = -\omega_{p_1} \]

\[ A(s) = \frac{a_o}{1 + T_o} \cdot \left[ \frac{s}{p_1 \cdot (1 - |a_o| \cdot f)} \right] \]
Effect of Feedback on Frequency Response (Cont.)

Since,

\[ T_o = -|a_o| \cdot f \]

<table>
<thead>
<tr>
<th>( a_o \cdot f = 1 )</th>
<th>( \omega_{p1} )</th>
<th>Stable</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_o \cdot f &lt; 1 )</td>
<td></td>
<td>Unstable</td>
</tr>
<tr>
<td>( a_o \cdot f &gt; 1 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

If \( T < -1 \) or \( (1+T) < 0 \) the circuit is unstable.
Effect of Feedback on Frequency Response (Cont.)

The condition for stability of a multipole response is the Nyquist Criteria.

$$A(s) = \frac{a(s)}{1 + a(s) \cdot f} = \frac{a(s)}{1 + T(s)}$$

Simple Version:
If $|T(j\omega)| > 1$ at the frequency where the phase of $T(j\omega) = -180^\circ$, then the circuit is unstable.

$$T(j\omega) = T(s)|_{s=j\omega}$$

$$\theta_{T(j\omega)} = arc\tan\left[\frac{Im\{T(j\omega)\}}{Re\{T(j\omega)\}}\right]$$
Effect of Feedback on Frequency Response (Cont.)

Complex Nyquist Criteria:

Plot $T(j\omega)$ on complex plane. As $\omega$ increases count number of times $-1$ is circled - even number means unstable (I Think).

3 - Poles

$\omega < 0$

$\omega = 0$

$\omega > 0$

$\omega \to \infty$
Effect of Feedback on Frequency Response (Cont.)

\[ v_{IN}(\omega) \rightarrow \Sigma \rightarrow a(\omega) \rightarrow v_{o}(\omega) \]

\[ |T(j\omega)|_{180^\circ} < 1 \]

Worst Case Stability Condition
Effect of Feedback on Frequency Response (Cont.)

$|T(j\omega)|_{dB}$

- $f = 0.1$
- $f = 1$
- $T(\omega) = a(\omega)$

Frequency where $\theta = -180^\circ$

$|T(\omega)| = 1$

$\theta = 180^\circ$

$0.1\omega_p$

$\omega_p$

$\omega_{p2}$

$\omega_{p3}$

$0dB$

$20dB$

$-90^\circ$

$-180^\circ$

$-270^\circ$
PHASE MARGIN : Difference between the actual phase shift and \(-180^\circ\)

when \( |T(\omega)| = 1 \)

i.e. \( \theta_m \equiv \text{Phase Margin} = \theta[T(\omega)] - (-180^\circ) \)

if \( \theta_m > 0 \) then the amplifier is stable - typically \( 45^\circ - 60^\circ \)

\[
A = \frac{1}{f} \quad \text{more gain more stable}
\]

\[
R_{OUT} = \frac{r_o}{1 + T} \quad \text{higher } R_{OUT} \text{ with more gain}
\]
Effect of Feedback on Frequency Response (Cont.)

Always Stable

Worst Case for Stability
As $\theta_m$ approaches 0 the amplifier is becoming unstable.

$|A(\omega)|_{dB}$
closed loop gain

$(A_o)_{dB}$
Effect of Feedback on Frequency Response (Cont.)

\[ A(\omega) = \frac{a(\omega)}{1 + a(\omega) \cdot f} \]

\[ a(s) = \frac{N(s)}{D(s)} \]

\[ A(s) = \frac{\frac{N(s)}{D(s)}}{1 + \frac{N(s)}{D(s)} \cdot f} = \frac{N(s)}{D(s) + N(s) \cdot f} \]

if the feedback factor is frequency dependent, then,

\[ f(s) = \frac{N_f(s)}{D_f(s)} \]

\[ A(s) = \frac{N(s)D(s)}{D(s)D_f(s) + N(s)N_f(s)} \]
Compensation is the method in which an amplifier is modified so that it is stable.

One way is to decrease $f$ (less feedback).

If $\omega_{180}$ is the frequency where,
\[
\theta(a(\omega_{180})) = -180^\circ
\]
then if,
\[
f < \left| \frac{1}{a(\omega_{180})} \right|
\]
then,
\[
|T(\omega_{180})| = f \cdot |a(\omega_{180})|
\]
and stability is ensured.
Narrowbanding for Compensation

This entails the addition of a dominant pole

\[ \frac{\omega_p}{a_o \cdot f} = \omega_c \]

\[ \theta_m = 45^\circ \]

\[ \omega_{180^\circ} \]

\[ -135^\circ \text{ or } \theta_m = 45^\circ \]
Narrowbanding for Compensation (Cont.)

For,

$$\theta_m = 45^\circ$$

add a compensation pole, $\omega_c$ at the frequency,

$$\frac{\omega_p}{|a_{of}|} = \omega_c$$

$$\omega_p = 1 MHz$$

$$|a_{of}| = 10^4$$

$$\omega_c = 100 Hz$$

$-90^\circ$ of phase shift from the new compensation pole.

$-45^\circ$ from the second pole.
Pole Splitting

It is better to use an existing pole rather than add another.

\[
\frac{a_1}{1 + j \frac{\omega}{\omega_{p1}}} \quad \frac{g_m R_L}{\left(1 + j \frac{\omega}{\omega_{p2}}\right) \cdot \left(1 + j \frac{\omega}{\omega_{p3}}\right)} \quad \frac{a_2}{1 + j \frac{\omega}{\omega_{p4}}}
\]
Pole Splitting (Cont.)

Let's say $\omega_{p1}$ and $\omega_{p4}$ are given with,

$$\omega_{p4} \gg \omega_{p1}$$

$$C_{GD} \ll C_{GS}, C_D$$

then,

$$\omega_{p2} = \frac{1}{R_{DIFF} C_{GS}}$$

$$\omega_{p3} = \frac{1}{R_L C_D}$$

$$\omega_z = \frac{g_m}{C_{GD}}$$

after $C_C$

$\omega_{p2}$ with $C_C$

$\frac{g_m}{C_{GD}}$
\[ |A(\omega)|_{dB} \]

Closed loop gain

\[(A_O)_{dB} \quad \text{< 3dB} \]

\[ t_{\text{RISE}} + t_{\text{FALL}} \]

\[ t_{\text{SETTLE}} = \frac{t_{\text{RISE}} + t_{\text{FALL}}}{2} \]
Pole Splitting (Cont.)

If we add a compensation capacitor, $C_c$ in parallel with $C_{GD}$:

$$\omega_{p2} = \frac{1}{R_{DIFF} \cdot (1 + g_m \cdot R_L) \cdot C_c}$$

$$\omega_{p3} = \frac{g_m}{C_{GS} + C_D}$$

Lets put numbers in:

$$R_{DIFF} = 10 \text{ M} \Omega$$
$$R_L = 5 \text{ M} \Omega$$
$$C_{GS} = 0.1 \text{ pF}$$
$$C_D = 0.1 \text{ pF}$$
$$g_m = 10^{-3} \text{ Mhos}$$

$$\omega_{p1} = 10 \cdot 10^6 \text{ rad/sec}$$
$$\omega_{p4} = 100 \cdot 10^6 \text{ rad/sec}$$

$$a_1 = 10^3$$
$$a_2 = 1$$
Before compensation, and with,

\[ C_{GD} = 0 \]

\[ \omega_{p2} = \frac{1}{10^7 \cdot 10^{-13}} = 10^6 \frac{rad}{sec} \]

\[ \omega_{p3} = \frac{1}{5 \cdot 10^6 \cdot 10^{-13}} = 2 \cdot 10^6 \frac{rad}{sec} \]

\[ a(\omega) = \left( \frac{10^3}{1 + j\omega} \right) \cdot \left( \frac{10^{-3} \cdot 5 \cdot 10^6}{1 + j\omega} \right) \cdot \left( \frac{1}{1 + j\omega} \right) \]
Compensate this amplifier for the worst case,

\[ f = 1 \]

with,

\[ \theta_m = 45^\circ \]
Somewhere between 2MHz and 10MHz,
\[ \theta = -180^\circ \]
but the loop gain,
\[ T \gg 1 \quad T \sim 10^5 \]
So how to compensate it?
Add \( C_c \) so that the gain at the first non-dominant pole (\( w_{p1} \)). since \( w_{p3} \) will move to a higher frequency and \( w_{p2} \) will move lower.
Pole Splitting (Cont.)

\[ \omega_{p_2} = \frac{\omega_{p_1}}{5 \times 10^6} = \frac{10^7}{5 \times 10^6} = 2 \cdot \frac{rad}{sec} \]

Formula for \( \omega_{p_2} \) & \( \omega_{p_3} \) with \( C_C \):

\[ C_C \gg C_{GS}, C_D \]

\[ \omega_{p_2} = \frac{1}{R_{DIFF} \cdot (1 + g_m \cdot R_L) \cdot C_C} \]

\[ \omega_{p_3} = \frac{g_m}{C_{GS} + C_D} \]

\[ \omega_z = \frac{g_m}{C_C} \]
Pole Splitting (Cont.)

\[ C_c = 10 \text{pF} \]

\[ \omega_{p_2} = \frac{1}{10^7 \times 5 \times 10^3 \times C_c} \]

\[ \omega_{p_3} = \frac{10^{-3}}{0.2 \times 10^{-12}} = 5 \times 10^9 \cdot \text{rad/sec} = 5000 \text{Mrad/sec} \]

\[ \omega_z = \frac{10^{-3}}{10^{-11}} = 10^8 \cdot \frac{\text{rad}}{\text{sec}} \]

So by adding a 10pF capacitor this circuit is made stable.
Slew Rate Limited
Slew Rate & Compensation Miller Op Amp

\[ \frac{v_{id}}{2} \]

\[ -\frac{v_{id}}{2} \]

\[ I_{ss} \]

\[ M1 \]

\[ M2 \]

\[ I_{ss} \]

\[ C_C \]

Output Stage

\[ A_v = 1 \]

\[ v_o \]

\[ I_{ss} \]

\[ G_m \]

\[ v_{cd} \]

\[ -I_{ss} \]
Slew Rate & Compensation Miller Op Amp (Cont.)

Slew Rate Volts/µsec

- 10 low
- 20 – 50 med
- 100 high

Few 100mV
Slew Rate & Compensation Miller Op Amp (Cont.)

Circuit situation with large $v_{id}$

$$I_{ss} = -C_c \cdot \frac{dv_o}{dt} \quad \text{or} \quad \frac{dv_o}{dt} = \frac{-I_{ss}}{C_c} = \text{slew rate}$$

$$v_o = \frac{1}{C} \cdot \int I_{ss} \cdot dt = \frac{I_{ss}}{C} \cdot t \quad \leftarrow \text{linear with time}$$
Slew Rate & Compensation Miller Op Amp (Cont.)

\[ T(\omega) \]

\[ a_o \]

\[ \omega_{p1} \]

\[ \omega_{p2} \]

\[ \theta_m = 45^\circ \]

\[ f = 1 \]

\[ a_o = a_{DIFF} \cdot a_g \]

Location of compensation pole:

\[ \omega_c = \frac{\omega_{p2}}{a_o} \]
Slew Rate & Compensation Miller Op Amp (Cont.)

\[ a_{\text{DIFF}} = g_m \cdot R_{D,\text{DIFF}} \]

\[ \omega_C = \frac{1}{R_{D,\text{diff}} \cdot a_g \cdot C_C} = \frac{\omega_{p2}}{a_o} = \frac{\omega_{p2}}{a_{\text{DIFF}} \cdot a_g} \]

\[ \frac{\omega_{p2}}{a_{\text{DIFF}}} = \frac{1}{R_{D,\text{DIFF}} \cdot C_C} = \frac{\omega_{p2}}{g_m \cdot R_{D,\text{DIFF}}} \]

\[ C_C = \frac{g_m}{\omega_{p2}} \text{ The size of the compensation depends only on } g_m \text{ & } \omega_{p2} \]
Slew Rate & Compensation Miller Op Amp (Cont.)

\[ g_m = \frac{2 \cdot I_{DS}}{V_{DSAT}} \]

\[ V_{DSAT} = \frac{2 \cdot I_{DS}}{g_m} \]

\[ g_{m1} \bigg|_{I_{DS} = \frac{I_{SS}}{2}} \]

\[ \frac{dv_o}{dt} = \frac{I_{SS}}{C_C} = \frac{I_{SS}}{g_{m1}} \cdot \omega_{p2} = \frac{I_{SS} \cdot 2}{g_{m1}} \cdot \omega_{p2} \]

**Slewrate** = \( \frac{dv_o}{dt} = V_{DSAT1} \cdot \omega_{p2} \)

\[ V_{DSAT} = 0.1 \text{V} \]

\[ \omega_{p2} = 10 \text{MHz} \cdot 2\pi \quad \text{SR} = 6.3 \text{V/\mu sec} \]
Slew Rate & Compensation Miller Op Amp (Cont.)

How to increase slew rate:

Increase $V_{DSAT_1}$ ⇒ More current, smaller $\frac{W}{L}$

Increase $\omega_{p2}$

Slew rate limits max change:

$$\frac{dV_{SIG}}{dt} = \omega_s \cdot A \cos \omega_s \cdot t$$

Max Value $\omega_s \cdot A$

$A = 2V \quad \omega_s = 10^6 \cdot 2\pi$

$SR = 13V/\mu\text{sec}$
MOS Miller Amp - Right Half Plane Zero

\[ C_c \]
\[ C_1 \]
\[ R_{o, \text{diff}} \]
\[ i_{\text{diff}} \]
\[ v_1 \]
\[ v_o \]
\[ g_m v_1 \]
\[ R_{\text{OUT, GAIN}} \]

ignore for now

also

removes zero entirely
MOS Miller Amp - Right Half Plane Zero (Cont.)

$$\omega_z = \frac{1}{C_C \cdot \left( \frac{1}{g_m} - R_Z \right)}$$

Diagram:

- A gain of $\frac{g_m}{C_C}$.
- A gain of $\frac{1}{C_C \cdot \left( \frac{1}{g_m} - R_t \right)}$.
- A gain of $\infty$. 

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