Effect of Feedback on Frequency Response

Let $a(\omega)$ be a single pole response,

$$a(s) = \frac{a_o}{1 - \frac{s}{p_1}} \Rightarrow a(\omega) = \frac{a_o}{1 + \frac{j\omega}{\omega_p}}$$

$$p_1 = -\omega_p$$

$$\frac{v_{out}(s)}{v_{in}(s)} = A(s) = \frac{a(s)}{1 + a(s) \cdot f} = \frac{1}{f} \left( \frac{T(s)}{1 + T(s)} \right)$$

$$A(s) = \frac{1 - \frac{s}{p_1}}{1 + \frac{a_o}{p_1} \cdot f} = \frac{a_o}{1 + a_o \cdot f} \left( \frac{1}{p_1 - (1 + a_o \cdot f)} \right)$$
Effect of Feedback on Frequency Response (Cont.)

Let $T_o = a_o \cdot f$

$$A(s) = \frac{a_o}{1 + T_o} \left( \frac{1 - \frac{s}{p_i \cdot (1 + T_o)}}{1 - \frac{s}{p_i}} \right)$$

Pole is at $p_i \cdot (1 + T_o) \Rightarrow -\omega_p \cdot (1 + T_o)$

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Gain reduction by negative feedback reduces Gain by $\frac{1}{1 + T_o}$ and increases bandwidth by $(1 + T_o)$

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\[\text{Effect of Feedback on Frequency Response (Cont.)} \quad \text{SB-5}\]

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\[\text{Effect of Feedback on Frequency Response (Cont.)} \quad \text{SB-6}\]

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\[\text{Effect of Feedback on Frequency Response (Cont.)} \quad \text{SB-7}\]
Effect of Feedback on Frequency Response (Cont.)

Since,
\[ T_\omega = -|a| \cdot f \]

\[ |a| \cdot f = 1 \]

Stable

Unstable

\[ |a| \cdot f > 1 \]

pole is at \( -\omega_p \cdot (1 - |a| \cdot f) \)

If \( T < -1 \) or \((1+T) < 0\) the circuit is unstable

Effect of Feedback on Frequency Response (Cont.)

The condition for stability of a multipole response is the Nyquist Criteria.

\[ A(s) = \frac{a(s)}{1 + a(s) \cdot f} = \frac{a(s)}{1 + T(s)} \]

Simple Version:

If \(|T/j\omega| > 1\) at the frequency where the phase of \( T(j\omega) = -180^\circ \), then the circuit is unstable.

\[ T(j\omega) = T(s) \big|_{s=j\omega} \]

\[ \theta_{s=j\omega} = \arctan \left[ \frac{\text{Im}(T(j\omega))}{\text{Re}(T(j\omega))} \right] \]

Effect of Feedback on Frequency Response (Cont.)

Complex Nyquist Criteria:

Plot \( T(j\omega) \) on complex plane. As \( \omega \) increases count number of times \(|T(j\omega)|\) circles \(-1\) is circled - even number means unstable (I think).

Effect of Feedback on Frequency Response (Cont.)

\[ |T(j\omega)|_\infty < 1 \]

Worst Case Stability Condition
**Effect of Feedback on Frequency Response (Cont.)**

PHASE MARGIN: Difference between the actual phase shift and $-180^\circ$

when $|\Gamma(\omega)| = 1$

i.e. $\theta_m = \text{Phase Margin} = \theta[T(\omega)] - (-180^\circ)$

if $\theta_m > 0$ then the amplifier is stable - typically $45^\circ - 60^\circ$

$A = \frac{1}{f}$ more gain more stable

$R_{out} = \frac{r_o}{1 + T}$ higher $R_{out}$ with more gain
Effect of Feedback on Frequency Response (Cont.)

\[ A(\omega) = \frac{a(\omega)}{1 + a(\omega) \cdot f} \]

\[ a(s) = \frac{N(s)}{D(s)} \]

\[ A(s) = \frac{N(s)D(s)}{1 + \frac{N(s)}{D(s)} \cdot f} = \frac{N(s)}{D(s) + N(s) \cdot f} \]

zeros of \( a(s) \)

poles of \( a(s) \)

if the feedback factor is frequency dependent, then,

\[ f(s) = \frac{N(s)}{D(s)} \]

\[ A(s) = \frac{N(s)D(s)}{D(s)D(s) + N(s)N(s)} \]

Compensation

Compensation is the method in which an amplifier is modified so that it is stable.

One way is to decrease \( f \) (less feedback).

If \( \omega_{180} \) is the frequency where,

\[ \theta(a(\omega_{180})) = -180^\circ \]

then if,

\[ f < \left| \frac{1}{a(\omega_{180})} \right| \]

then,

\[ |T(\omega_{180})| = f \cdot |\omega_{180}| \]

and stability is ensured.

Narrowbanding for Compensation

This entails the addition of a dominant pole

\[ |a_f|, \omega_c, \omega_p, \omega_{180} \]

\[ |T(j\omega)|_{dB} \]

\[ -90^\circ, -180^\circ, -270^\circ \]

\[ 0dB \]

\[ \omega_{180} = \omega_c \]

\[ \omega_{180} - f = \omega_c \]

\[ \theta = 45^\circ \]

\[ \omega_p, \theta = 45^\circ \]

\[ \omega = 1 MHz \]

\[ |\omega_f| = 10^4 \]

\[ \omega_c = 100Hz \]

\[ -90^\circ \] of phase shift from the new compensation pole.

\[ -45^\circ \] from the second pole.

Narrowbanding for Compensation (Cont.)

For,

\[ \theta_c = 45^\circ \]

add a compensation pole, \( \omega_c \) at the frequency,

\[ \frac{\omega_c}{\omega_f} \]

\[ \omega_c = 1 MHz \]

\[ |\omega_f| = 10^4 \]

\[ \omega_c = 100Hz \]
Pole Splitting

It is better to use an existing pole rather than add another.

\[
\begin{align*}
\frac{a_1}{1 + j \frac{\omega}{\omega_{p1}}} &= \frac{g_m R_l}{1 + j \frac{\omega}{\omega_{p1}}} \\
\frac{a_2}{1 + j \frac{\omega}{\omega_{p2}}} &= \frac{g_m R_l}{1 + j \frac{\omega}{\omega_{p4}}}
\end{align*}
\]

Pole Splitting (Cont.)

Let's say \(\omega_{p1}\) and \(\omega_{p4}\) are given with,

\[
\begin{align*}
\omega_{p2} &= \omega_{p1} \\
C_{GD} C_{GS} C_D
\end{align*}
\]

then,

\[
\begin{align*}
\omega_{p1} &= \frac{1}{R_{ref} C_{GD}} \\
\omega_{p4} &= \frac{1}{R C_D}
\end{align*}
\]

Pole Splitting (Cont.)

If we add a compensation capacitor, \(C_c\) in parallel with \(C_{GD}\):

\[
\omega_{p2} = \frac{1}{R_{ref} \cdot (1 + g_m \cdot R_l) \cdot C_c}
\]

\[
\omega_{p4} = \frac{g_m}{C_{GD} + C_c}
\]

Let's put numbers in:

\[
\begin{align*}
R_{ref} &= 10 \text{ Meg}\Omega \\
R_l &= 5 \text{ Meg}\Omega \\
C_{GD} &= 0.1 \text{ pF} \\
C_c &= 0.1 \text{ pF} \\
g_m &= 10^{-3} \text{ Mhos} \\
a_1 &= 10^{-3} \text{ rad sec} \\
a_2 &= 10^{-3} \text{ rad sec}
\end{align*}
\]
Before compensation, and with,

\[ C_{id} = 0 \]

\[ \omega_{p1} = \frac{1}{10^3 \cdot 10^6} = 10^{-7} \text{ rad sec} \]

\[ \omega_{p2} = \frac{1}{5 \cdot 10^{-7} \cdot 10^6} = 2 \cdot 10^{-7} \text{ rad sec} \]

\[ a(\omega) = \left( \frac{10^4}{1 + j \frac{\omega}{4 \cdot 10^6}} \right) \cdot \left( \frac{10^4 \cdot 5 \cdot 10^6}{1 + j \frac{\omega}{10^6}} \cdot \left( 1 + j \frac{\omega}{2 \cdot 10^6} \right) \cdot \left( 1 + j \frac{\omega}{10^6} \right) \right) \]

Compensate this amplifier for the worst case, \( f = 1 \) with, \( \theta_m = 45^\circ \)

Somewhere between 2MHz and 10MHz,

\[ \theta = -135^\circ \]

\[ \theta = -225^\circ \]

but the loop gain,

\[ T = 1 \quad T \sim 10^4 \]

So how to compensate it? Add \( C_c \) so that the gain at the first non-dominant pole (\( \omega_{p1} \)). since \( \omega_{p3} \) will move to a higher frequency and \( \omega_{p2} \) will move lower.

Formula for \( \omega_{p2} \) with \( C_c \):

\[ \omega_{p2} = \frac{\omega_{p1}}{5 \times 10^7} = \frac{10^4}{5 \times 10^7} = 2 \cdot \frac{\text{rad}}{\text{sec}} \]

\[ C_c = C_{id} \quad C_g \]

\[ \omega_{p2} = \frac{1}{R_{in} \cdot (1 + g_m \cdot R_1) \cdot C_c} \]

\[ \omega_{p3} = \frac{g_m}{C_{id} + C_{po}} \]

\[ \omega_s = \frac{g_s}{C_c} \]
Pole Splitting (Cont.)

\[ C_c = 10pF \]
\[ \omega_{p1} = \frac{1}{10^7 \times 5 \times 10^7 \times C_c} \]
\[ \omega_{p1} = \frac{10^{-7}}{0.2 \times 10^{-7}} = 5 \times 10^7 \text{ rad/sec} = 5000 \text{Mrad/sec} \]
\[ \omega_s = \frac{10^7}{10^{10}} = 10^7 \text{ rad/sec} \]

So by adding a 10pF capacitor this circuit is made stable.
Slew Rate & Compensation Miller Op Amp (Cont.)

Slew Rate Volts/µsec:
- 10 low
- 20 - 50 medium
- 100 high

Circuit situation with large \( \nu_o \)\

\[ I_{ss} = -C_v \frac{d\nu_o}{dt} \quad \text{or} \quad \frac{d\nu_o}{dt} = -\frac{I_{ss}}{C_v} = \text{slew rate} \]

\[ \nu_o = \frac{1}{C} \int I_{ss} \cdot dt = \frac{I_{ss}}{C} \cdot t \quad \rightarrow \quad \text{linear with time} \]

Location of compensation pole:
\[ \omega_c = \frac{\omega_p}{a_s} \]

\[ \theta_c = 45^\circ \]

\[ f = 1 \]

\[ a_s = a_{diff} \cdot a_c \]

\( a_{diff} = g_m \cdot R_{diff} \)

\[ \frac{1}{R_{diff} \cdot a_s \cdot C_c} = \frac{\omega_p}{a_s} \cdot \frac{\omega_p}{a_{diff} \cdot a_c} \]

\[ \frac{\omega_p}{a_{diff}} = \frac{1}{R_{diff} \cdot C_c} = g_m \cdot R_{diff} \]

\[ C_c = \frac{R_m}{\omega_c} \quad \text{The size of the compensation depends only on } g_m \text{ and } \omega_c \]
Slew Rate & Compensation Miller Op Amp (Cont.)  

\[
g_m = \frac{2 \cdot I_{ss}}{V_{dsat}}
\]

\[
V_{dsat} = \frac{2 \cdot I_{ss}}{g_m}
\]

\[
g_m \left( \frac{1}{\omega_p^2} \right)
\]

\[
\frac{dv_c}{dt} = \frac{I_{ss}}{C_c} = \frac{I_{ss}}{g_m \cdot \omega_p} = \frac{2I_{ss}}{g_m} \cdot \omega_p
\]

Slew rate:

\[
SR = \frac{dv_c}{dt} = V_{dsat} \cdot \omega_p
\]

\[
V_{dsat} = 0.1\,V
\]

\[
\omega_p = 10\,MHz \cdot 2\pi
\]

\[
SR = 6.3\,V/\mu\text{sec}
\]

How to increase slew rate:

Increase \( V_{dsat} \rightarrow \) More current, smaller \( \frac{W}{L} \)

Increase \( \omega_p \)

Slew rate limits max change:

\[
A \sin \omega_s \cdot t
\]

Max rate of change here

\[
A = 2V \quad \omega_s = 10\cdot 2\pi
\]

\[
SR = 13V/\mu\text{sec}
\]

Max Value \( \omega_s \cdot A \)

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MOS Miller Amp - Right Half Plane Zero

\[
\omega_s = \frac{1}{C_c \cdot \left( \frac{1}{g_m} - R_s \right)}
\]

\[
\frac{1}{C_c \cdot \left( \frac{1}{g_m} - R_s \right)}
\]

Also removes zero entirely