1 Schmitt Trigger

Consider the circuit below. The inverter is ideal, with \( V_M = \frac{V_{DD}}{2} \) and infinite slope. The transistors have \( |V_T| = 0.7 \text{V} \), \( k_n = 20 \ \mu\text{A/V}^2 \) and \( k_p = 8 \ \mu\text{A/V}^2 \). M1 has \((W/L) = 1\). Ignore all other parasitic effects in the transistors.

1A As \( V_{IN} \) goes from 0 to \( V_{DD} \) and back to 0 explain the sequence of events which makes this circuit operate as a Schmitt Trigger.

1B Find the value of \((W/L)_2\) such that when \( V_{IN} \) increases from 0 to \( V_{DD} \) the output will switch at \( V_{IN} = 3\text{V} \).

1C Find the value of \((W/L)_3\) such that when \( V_{IN} \) decreases from \( V_{DD} \) to 0 the output will switch at \( V_{IN} = 2\text{V} \). If you don’t trust your value from b., you may use \((W/L)_2 = 5\).
2 Oscillator

For the oscillator below, determine the oscillation frequency and draw the waveforms in nodes X, Y and Z. You may assume that the delay of the inverters, the resistances of the MOS transistors and all internal capacitances can be ignored. The inverters switch at $V_{IN} = V_{DD}/2$. Assume that nodes Y and Z are initially at 0 and $V_{DD}$, respectively.
3 Ling Adder

In this problem you will analyze a carry-lookahead adder proposed by Huey Ling (IBM) more than 20 years ago, but still among the fastest adders available.

In a **conventional adder**, in order to add two numbers:

\[
A=a_{n-1}2^{n-1} + a_{n-2}2^{n-2} + \ldots + a_02^0 \\
B=b_{n-1}2^{n-1} + b_{n-2}2^{n-2} + \ldots + b_02^0
\]

we first compute the local carry generate and propagate terms:

\[
g_i = a_i b_i \\
p_i = a_i + b_i
\]

then, with a ripple or a tree circuit we form the global carry-out terms resulting from the recurrence relation:

\[
G_i = g_i + p_i G_{i-1} \quad (1)
\]

Finally, we form the sum of A and B using local expressions:

\[
S_i = p_i \oplus G_{i-1} \quad (2)
\]

In the conventional adder, the terms \(G_i\) have, as described, a physical significance. However, an arbitrary function could be propagated, as long as sum terms could be derived. **Ling's** approach is to replace \(G_i\) with:

\[
H_i = G_i + G_{i-1}
\]

i.e. \(H_i\) is true if “something interesting happens at bit \(i\)” – there is a carry out or a carry in. \(H_i\) is called “Ling's pseudo-carry”.

3A **Prove that:** \(H_i = g_i + t_i H_{i-1}\) \quad (3)

where \(t_i = a_i + b_i\) (Ling had the bad idea to change some of the notations, but everybody uses his notations for describing his adder).

3B **Find a formula for computing the sum out of the operands and Ling’s pseudo-carry.**

3C **Unroll the recursions described by equations (1) and (3) for i=3.** You should get the expressions of \(G_3\) and \(H_3\) as a function of the bits of input operands. Simplify the expressions as much as possible.

3D **Implement the two functions using n-type dynamic gates.** Draw the two gates and size the transistors. Which one helps us build a faster adder? Explain your answer.
4 Mystery datapath

This circuit implements a 1-bit datapath function in dynamic logic.

4A Write down the boolean expressions for outputs F and G. On which clock phases are outputs F and G valid? When are the inputs (A, B, C_in) allowed to change (so that the circuit works properly)?

4B To what datapath function could this unit be most directly used (e.g. addition, subtraction, shifting)?

4C What is the purpose of transistor M1?

4D How can be evaluation phase of F be sped up by rearranging transistors? (no transistors should be added, deleted or resized)

4E Can evaluation of G be sped up in the same manner? Explain your answer.