1 Equivalent resistance

1A Consider the fictive device whose I-V characteristic is given in Figure 1. Constant k is a conductance in Siemens (or 1/Ω). Determine the equivalent resistance equation (as a function of k) over the transition from V=[0, 2V₀].

\[ I = kV^e^{(V/V₀)} \]

![Figure 1: Fictive device with non-linear I-V curve.](image)

1B The goal of this part is to obtain the resistance equation 3.42 of the textbook using the setup shown in Figure 2 below. Assuming the V₆S of the MOSFET is kept at V_DD, calculate the R_eq as the output (V_DS = V_OUT) transitions from V_DD to V_DD/2. Make sure that you use the short-channel, unified MOS equations (textbook Figure 3-23).

*Hint #1: You will need to use the expansion: ln(1+x) ≈ x - x^2/2 + x^3/3*

*Hint #2: You may find the following property useful: \[ \frac{u}{1+u} = 1 - \frac{1}{1+u} \]

![Figure 2. MOSFET discharging a capacitor](image)
1C Consider the circuit in the Figure 3. Before time t=0 the switch is open and Vout is charged up to some $V_{OH}$. At time t=0, switch closes and output starts dropping. The switch has a nonzero, but small, on resistance. Use $W/L = 0.25\mu/0.25\mu$, so that the short channel models are valid. Use the following transistor parameters:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>NMOS</th>
<th>PMOS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>0 V$^{-1}$</td>
<td>0 V$^{-1}$</td>
</tr>
<tr>
<td>$V_{T0}$</td>
<td>0.43 V</td>
<td>-0.4 V</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.4 V$^{0.5}$</td>
<td>-0.4 V$^{0.5}$</td>
</tr>
<tr>
<td>$k'$</td>
<td>115 uA/V$^2$</td>
<td>-30 uA/V$^2$</td>
</tr>
<tr>
<td>$V_{DSAT}$</td>
<td>+inf</td>
<td>-inf</td>
</tr>
</tbody>
</table>

ie. neglect the channel length modulation

ie. neglect velocity saturation

Complete the following:

i. The average of the highest and lowest $V_{th}$ values during the transition.

ii. Calculate the output voltage $V_{OH}$ right before time t=0.

iii. Calculate the equivalent resistance during the $V_{OH} \rightarrow V_{OH}/2$ transition by integrating the $R(v)$

iv. Recalculate the equivalent resistance during the $V_{OH} \rightarrow V_{OH}/2$ transition by averaging the resistance at the two end points.

v. Explain why the two methods of calculating equivalent resistance mismatch.

Note: If you encounter an infinite resistance (ie. the transistor is off), instead use the resistance at the point the transistor turns ‘on’.

Figure 3. Circuit that discharges capacitor through a switched resistance.
2 Propagation Delay.

Consider the inverter layout shown in Figure 6. Assume it is a 0.25u CMOS process and that the grid steps are 0.25u.

2A When this inverter drives another inverter of the same size, calculate the total effective capacitance ($C_z$) on the intermediate node. (i.e Node Z) for the output transitions Vout: $V_{DD} \rightarrow V_{DD}/2$ and $0 \rightarrow V_{DD}/2$. Note that $L = 0.25u$ for all transistors. Use the capacitance parameters given in Table 3.5 of the textbook.

![Figure 4](image)

2B Calculate the $R_{peq}$ and $R_{neq}$, ie. the equivalent resistances for NMOS and PMOS devices.

2C Calculate $t_{pHL}$, $t_{plH}$, $t_p$.

2D OPTIONAL (not graded): simulate this inverter pair in SPICE to verify your calculations. Make sure that you include the source and drain areas and perimeters in your transistors, e.g.:

```
M1 D G S B modelname L=xu W=yu AD=ad AS PD=drnper PS=srcper
```
3 Inverter in Weak Inversion Regime

The inverter in Figure 5 operates in the weak inversion regime with $V_{DD}=0.4\text{V}$. Use the following parameters:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>NMOS</th>
<th>PMOS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>$0.06 \text{ V}^{-1}$</td>
<td>$-0.1 \text{ V}^{-1}$</td>
</tr>
<tr>
<td>$kT/q$</td>
<td>26 mV</td>
<td>26 mV</td>
</tr>
<tr>
<td>$n$</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>$I_S$</td>
<td>$I_S$</td>
<td>$-I_S$</td>
</tr>
</tbody>
</table>

Note: also called $V_T$ (thermal voltage)

Use the subthreshold voltage current equation in the textbook (eqn 3.39).

3A Calculate the switching threshold $V_M$ (ie. when $V_{OUT} = V_{IN}$) of this inverter.

3B Calculate $V_{IL}$ and $V_{IH}$ of the inverter.

Figure 5. Inverter schematic.
Figure 6. Inverter layout for problem 2; distances in lambda units (lambda = 0.125u); transistor L=2 lambda, Ldiff=6 lambda, Wp=20 lambda, Wn=10 lambda