First, we plot the I-V curves for M1 using SPICE.

Then, we transform the I-V curves for the fictitious device to the $I_D$ vs. $V_{OUT}$ coordinate axis, and superimpose the two graphs.

From the given I-V plot, we can locate a few points:

<table>
<thead>
<tr>
<th>$V$</th>
<th>$V_{OUT}$ = $V_{DD}$ - $V$</th>
<th>$I = I_D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.5</td>
<td>0</td>
</tr>
<tr>
<td>0.5</td>
<td>2</td>
<td>0.75 $\mu$</td>
</tr>
<tr>
<td>1</td>
<td>1.5</td>
<td>50 $\mu$</td>
</tr>
<tr>
<td>1.5</td>
<td>1</td>
<td>150 $\mu$</td>
</tr>
<tr>
<td>2</td>
<td>0.5</td>
<td>210 $\mu$</td>
</tr>
</tbody>
</table>

I've plotted these points and connected the dots.

Intersections represent possible operating points.

To find $V_{OL}$ and $V_{OH}$, recall the definition (or property) that $f(V_{OH}) = V_{OL}$ and $f(V_{OL}) = V_{OH}$.

We start by guessing $V_{OL} = V_{DD}$. From the plot this gives $V_{OL} \approx 0.73$ (#2) which gives $V_{OH} \approx 2.4\%$, which roughly gives $V_{OL} \approx 0.75$ (#3), which converges to $V_{OH} \approx 2.4\%$ (#4)

Therefore $V_{OL} \approx 2.4$ and $V_{OL} \approx 0.75$
2.

\[ \begin{align*}
V_m & \quad 2.5V \\
V_{in} & \quad 0V \\
\text{M1} & \quad W_C = 1.5 \\
\text{M2} & \quad W_C = 1.5 \\
\text{V}_{out} & \quad 0V \\
\end{align*} \]

a. Typical values are around \( V_{DNP} \), which is also a good starting guess for this circuit. More precisely, assume that \( V_m = 1.25 \) and that we have \( V_{in} = V_{Vsat} = 1.25 \).

Then \( V_{G_{S1}} = V_{G_{S2}} = V_{OS1} = V_{OS2} = 1.25 \), this cannot be a stable operating point since due to body effect \( V_{T2} \geq V_{T1} \) \( \Rightarrow \)

\( V_{GT2} \leq V_{GT1} \) \( \Rightarrow I_{D2} < I_{D1} \) (KCL requires \( I_{D1} = I_{D2} \)). To compensate, \( V_{out} \) will decrease until \( I_{D2} = I_{D1} \). Hence for \( V_{in} = 1.25 \), \( V_{out} < 1.25 \).

Both devices are saturated since \( V_{DS} > \sqrt{2} (V_{SAT}, V_{GT}) \), and M1 is velocity saturated since \( V_{GT2} > \frac{V_{SAT}}{1.25} \), but M2 is borderline, because \( \frac{1.25}{V_{GSAT}} \approx 0.63 \)

body effect may cause \( V_{T2} \leq V_{Vsat} \). We'll assume it is velocity saturated for now.

b. \( I_{D1} = \frac{K_N(W)}{L} \left( V_{in} - V_{T1} - \frac{V_{os1}}{2} \right) V_{Sat} \left( 1 + \lambda V_{out} \right) \)

\( I_{D2} = \frac{K_N(W)}{L} \left( V_{os2} - V_{os1} - \frac{V_{os2}}{2} \right) V_{Sat} \left( 1 + \lambda (V_{os2} - V_{out}) \right) \)
3. [R96] \[ V_m = r \left( \frac{V_{DD} - V_{TP}}{1 + r} \right) + V_{TN} \]

\[ r = \sqrt{\frac{k_p}{k_n}} \]

for \( V_{DD} = 2.5 \), \( |V_{TP}| = .4 \), \( V_{TN} = .43 \), \( k_p = -3.0 \)

\[ \frac{k_p}{k_n} = 115 \]

we have \( V_m \approx .99 \)

[r97] \[ V_m = \left( \frac{V_{TP} + V_{DSAT}}{2} \right) + r \left( V_{DD} + V_{TP} + \frac{V_{DSAT}}{2} \right) \]

\[ r = \frac{k_p V_{DSAT}}{k_n V_{DSAT}} \quad \text{(always positive)} \]

\[ V_m = 1.00 \]

From SPICE, \( V_m \approx .99 \). Hence both formulas give close results.

This is because \( V_{QP} = V_T - V_T = .56 \) for the NMOS, and \( .59 \) for the PMOS.

In both cases \( V_{QP} < V_{DSAT} \), so the devices are not velocity saturated, so the long channel model is accurate, but \( V_{QP} \) is still roughly in the neighborhood of \( V_{DSAT} \) so the short channel model is also fairly accurate.
C. Setting $V_h = V_{out} = V_m$, we have

$$I_{DI} = k_n I_1 \left( V_m - V_t - \frac{V_{SAT}}{2} \right) V_{SAT} \left( 1 + \lambda V_h \right)$$

$$I_{DL} = k_n I_2 \left( V_{DD} - V_m - V_t - \frac{V_{SAT}}{2} \right) V_{SAT} \left( 1 + \lambda (V_{DD} - V_m) \right)$$

Now using KCL and ignoring $\lambda$ (Robbey ignores this as well [R99, P150]), but including it wouldn't affect things much since $\lambda$ is small and $V_{DD} - V_m \approx V_h$, we have

$$I_{DI} = I_{DL}$$

$$\Rightarrow V_m - V_t - \frac{V_{SAT}}{2} = V_{DD} - V_m - V_t - \frac{V_{SAT}}{2} \quad \text{(remember $\frac{V_{SAT}}{2} = \frac{V}{2}$)}$$

$$\Rightarrow 2V_m = V_{DD} + V_t - V_t$$

$$\Rightarrow V_m = \frac{V_{DD} + V_t - V_t}{2}$$

1. $V_1 = V_0 + \delta \left[ (V_{B1} + 1.2V_{t1}) \frac{L}{W} - (V_{B1} + V_{t1}) \frac{L}{W} \right] \quad \left| V_{t1} \right| = 0.6, \quad V = 0.4 \quad V_{t1} = 0.43$

Guess $V_m$, solve for $V_t$, find new value of $V_m$, repeat.

$V_m = 1.25 \Rightarrow V_{t1} = 1.25 \Rightarrow V_t = 0.66 \Rightarrow V_m = 1.14$

$\Rightarrow V_t = 0.65 \Rightarrow V_h = 1.14$

So $V_h = 1.14$ and $V_t = 0.65$

For $M_1$, $V_{GT1} = 1.14 - 0.43 = 0.71 > V_{SAT} = 0.63$, and $V_{DD} > V_{SAT}$

$\Rightarrow M_1$ vel saturated

For $M_2$, $V_{GT2} = 2.5 - 1.14 - 0.65 = 0.71 > V_{SAT} = 0.63$, and $V_{DD} > V_{SAT}$

$\Rightarrow M_2$ vel saturated.
4. \( V_{DD} \quad V_M \quad I_D(V_M) \quad g \quad V_H \quad NM_H \quad NM_L = V_L \)

\[
2.5 \quad 1.00 \quad 2.96 \times 10^{-5} \quad -21.6 \quad 1.05 \quad 1.45 \quad .93
\]

\[
1.8 \quad .79 \quad 1.17 \times 10^{-5} \quad -54.7 \quad .80 \quad 1 \quad .77
\]

\[
1.0 \quad .56 \quad 1.50 \times 10^{-6} \quad -42.7 \quad .56 \quad .44 \quad .56
\]

where

\[
V_M = \frac{(V_{th} + V_{Ssat})}{2} + \frac{r}{1 + r} \left( \frac{V_{DD} + V_{IP} + V_{Ssat}}{2} \right)
\]

\[
I_D(V_M) = \frac{k_N}{L_n} \left( V_{GTH} - V_{Min} \right)^2 \left( 1 + \lambda_n V_H \right)
\]

\[
V_{GTH} = V_M - V_{TN}
\]

\[
V_B = V_M
\]

\[
V_{Min} = V_{GTH}
\]

\[
I_D(V_M) = \frac{1}{2} k_N \left( V_{GTH} \right)^2 \left( 1 + \lambda_n V_H \right)
\]

\[
= 8.6 \times 10^{-5} \left( V_M - .43 \right)^2 \left( 1 + .06 V_H \right)
\]

\[
g = \frac{-1}{I_D(V_M)} \frac{k_N V_{GTH} + k_P V_{Ssat}}{L_n - A_P}
\]

\[
= \frac{-1}{I_D(V_M)} \left( 6.40 \times 10^{-4} \right)
\]

\[
V_{IH} = \frac{V_H - V_M}{g}
\]

\[
NM_H = V_{DD} - V_{IH}
\]

\[
V_{IL} = \frac{V_H + V_{DD} - V_M}{g}
\]

SPICE

\[
V_{DD} \quad V_{IH} \quad NM_H \quad NM_L
\]

\[
2.5 \quad 1.05 \quad 1.45 \quad .88
\]

\[
1.8 \quad .80 \quad 1 \quad .70
\]

\[
1.0 \quad .48 \quad .52 \quad .46
\]

Results are quite close, surprisingly close, with decreasing \( V_{DD} \)

Discrepancy increases probably because for low \( V_{DD} \) the devices are not velocity saturated.

Note the equation for \( V_M \) and \( g \) assume velocity saturation (which occurs in no case) but \( I_{DH} \) is correct.
Notice the slope never even reaches -1!

This circuit is indeed an inverter, but it's not regenerative. (exercise: verify this).

Current X=1.15e+00
Current Y=1.15e+00
Derivative=8.00e-01

agrees well with hand calculations
Spice files for ee141 sp00 hw2
*cbc@eecs
*ee141 sp00 hw2 #1
*minimum size NMOS I-V curves.

vin vin 0
vd mld 0
m1 mld vin 0 0 NMOS l=.25u w=.375u

.dc vd 0 2.5 .01 sweep vin 0 2.5 .125
.options post=2 nomod
.lib '~/ee141/MODELS/g25.mod' TT
.end

*ee141 sp00 hw2 #2e
*nmos enhancement load inverter

vdd vdd 0 2.5
vin vin 0
m1 vout vin 0 0 NMOS l=.25u w=.375u
m2 vout vdd vdd 0 NMOS l=.25u w=.375u

.dc vin 0 2.5 .01
.options post=2 nomod
.lib '~/ee141/MODELS/g25.mod' TT
.end

*ee141 sp00 hw2 #3
*standard cmos inverter

vdd vdd 0 2.5
vin vin 0
m1 vout vin 0 0 NMOS l=.25u w=.375u
m2 vout vin vdd vdd PMOS l=.25u w=.375u

.dc vin 0 2.5 .01
.options post=2 nomod
.lib '~/ee141/MODELS/g25.mod' TT
.end

*ee141 sp00 hw2 #4
*VTCs for vdd=2.5, 1.8, 1.0
*note this circuit generates 1+2+3=6 .sw0 files; i'm not sure why
*right now. i left my hspice manuals at home :P

.param supply=2.5
vdd vdd 0 supply
vin vin 0
m1 vout vin 0 0 NMOS l=.25u w=.375u
m2 vout vin vdd vdd PMOS l=.25u w=.375u

.dc vin 0 2.5 .01
.options post=2 nomod
.lib '~/ee141/MODELS/g25.mod' TT
.alter
.param supply=1.8
.dc vin 0 2.5 .01
.alter
.param supply=1.0
.dc vin 0 2.5 .01
.end