3.1. \[ K_{eq} = \frac{-q_0^m}{(V_{high} - V_{low})(1-m)} \left[ (q_0 - V_{high})^{1-m} - (q_0 - V_{low})^{1-m} \right] \]  

(Note: \( V_{high} \geq |V_{low}| \))

\[ V_{dd} = 3q_0 \quad q_0 = 0.9 \]

\[ M = \frac{1}{2} \text{ (abrupt junction)} \quad M = \frac{1}{3} \text{ (linear or graded junction)} \]

<table>
<thead>
<tr>
<th></th>
<th>( V_{low} )</th>
<th>( V_{high} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>L-0H</td>
<td>(-0.5q_0)</td>
<td>(-1.5q_0)</td>
</tr>
<tr>
<td>K(_{eq})</td>
<td>0.77</td>
<td>0.56</td>
</tr>
<tr>
<td>K(_{pp})</td>
<td>0.56</td>
<td>0.77</td>
</tr>
</tbody>
</table>

N\(_{Mos}\) L-0H: \( V_{low} = 0 \), \( V_{high} = -1.5q_0 \)

\[ K_{eq} = \frac{-q_0^m}{(-1.5q_0)(1-m)} \left[ (2.5q_0)^{1-m} - q_0^{1-m} \right] \]

N\(_{Mos}\) H-0L: \( V_{low} = -1.5q_0 \), \( V_{high} = -3q_0 \)

\[ K_{eq} = \frac{-q_0^m}{(-1.5q_0)(1-m)} \left[ (4q_0)^{1-m} - (2.5q_0)^{1-m} \right] \]

P\(_{Mos}\) L-0H: \( V_{low} = -1.5q_0 \), \( V_{high} = -3q_0 \)

\[ K_{eq} = \frac{-q_0^m}{(-1.5q_0)(1-m)} \left[ (4q_0)^{1-m} - (2.5q_0)^{1-m} \right] \] (same equation as \( ** \))

P\(_{Mos}\) H-0L: \( V_{low} = 0 \), \( V_{high} = -1.5q_0 \)

Same eqn as \( ** \).

We can notice two trends:

1. Capacitance worsens with decreasing \( M \)
2. For an N\(_{Mos}\), capacitance is greater during the low to high transition; the opposite holds for the P\(_{Mos}\).
a. $\tau_{PLH} = 69 \text{ ReqP } C_{LH}$  \hspace{1cm} $\tau_{PHL} = 69 \text{ ReqN } C_{HL}$

ReqP and ReqN are calculated in problem 3a. below, or see [R99 p.164-165].

To quickly compute $C_{LH}$, $C_{HL}$ we refer to [R99, table 5.2, p.163].

\[
C_l = \left( C_{g1} + C_{g2} + C_{x1} + C_{x2} \right) + 3 \left( C_{g1} + C_{x2} \right)
\]

\[
\text{Intrinsic} \quad \text{Extrinsic}
\]

\[
\Rightarrow C_{LH} = 12.4 \text{ fF} \quad C_{HL} = 12.1 \text{ fF} \quad \text{(roughly equal)}
\]

\[
\Rightarrow \tau_{PLH} = 58 \text{ ps} \quad \tau_{PHL} = 72 \text{ ps}
\]

\text{SPICE}: \quad \tau_{PLH} = 62.7 \text{ ps} \quad \tau_{PHL} = 68.6 \text{ ps.}
b. The extrinsic capacitance remains as before, but we will have to recompute the intrinsic capacitances.

\[ C_{GS} = 3 \times (0.23\mu F) = 0.69\mu F \quad (C_{GS} \propto W) \]

\[ C_{GD} = 3 \times (0.61\mu F) = 1.83\mu F \quad (C_{GD} \propto W) \]

\[ C_{DS} = K_{eq} A_P C_{j1} + K_{f} n_{mp} P_{D} C_{j2} \]

with \[ A_P = (91/51) = 0.79 \mu m^2 \] and \[ P_{D} = 91 + 51 = 2.42 \mu m \]

\[ K_{eq} = 0.57 \quad (H \rightarrow L) \quad \text{or} \quad 0.49 \quad (L \rightarrow H) \quad \text{(see example 5.3, p. 180)} \]

\[ K_{f} n_{mp} = 0.61 \quad (H \rightarrow L) \quad \text{or} \quad 0.81 \quad (L \rightarrow H) \]

\[ C_{j1} = 2.8\mu F / \mu m^2 \quad \text{and} \quad C_{j2} = 2.8\mu F / \mu m^2 \quad \text{(eq. 4.44, table 3.5, p. 80)} \]

\[ \Rightarrow C_{stage} = 1.7 \mu F / \mu m, \quad C_{BS} = 1.2 \mu F \]

\[ C_{DB} = K_{eq} A_P C_{j1} + K_{f} n_{mp} P_{D} C_{j2} \]

with

\[ K_{eq} = 0.79 \quad (H \rightarrow L) \quad \text{or} \quad 0.59 \quad (L \rightarrow H) \]

\[ K_{f} n_{mp} = 0.61 \quad (H \rightarrow L) \quad \text{or} \quad 0.78 \quad (L \rightarrow H) \]

\[ A_P = 2.71 + 51 = 4.62 \mu m^2 \]

\[ P_{D} = 4.62 \mu m \]

\[ C_{DS} = 1.9\mu F / \mu m^2 \quad \text{and} \quad C_{DS} = 2.2\mu F / \mu m^2 \]

\[ \Rightarrow C_{DBL} = 0.11 \mu F / \mu m, \quad C_{DBL} = 0.15 \mu F / \mu m \]

\[ \Rightarrow C_{DH} = 0.69 + 1.83 + 1.7 + \frac{3.08}{2} + 3(0.76 + 2.28) = 16.42\mu F \]

\[ C_{DC} = 0.69 + 1.83 + 1.2 + 4.04 + 3(0.76 + 2.28) = 16.98\mu F \]

Finally, \[ R_{eq} \propto \frac{1}{W} \Rightarrow R_{eq} = \frac{8.68k\Omega}{3}, \quad P_{eq} = \frac{6.99kW}{3} \]

\[ \Rightarrow T_{DHE} = 26\mu s, \quad T_{DHC} = 26\mu s \]

SPIKE: \[ T_{DHE} = 30.8\mu s, \quad T_{DHC} = 91.8\mu s \]

Our estimates are typically optimistic; this is partly because we are not accounting for overshoot.
3.3. \[ T_{PLH} = 0.69 \, R_{eq} \, C_L \] (5.19) \[ T_{PHL} = 0.69 \, R_{eq} \, C_L \] (5.18)

where \[ R_{eq} = \frac{3}{4} \frac{V_{DD}}{I_{DSAT}} \left(1 - \frac{3}{4} \frac{V_{DD}}{V_{DD}}\right) \]

(eq. 5.17)

\[ \lambda_p = 0.6 \, \lambda_n, \lambda_n = -1 \, \lambda_p, V_{DSAT} = 0.63 \, V, V_{DSS} = 1 \]

\[ K_{n} = 115 \times 10^{-6} \, A/V^2 \]

\[ V_{n} = 0.3 \, V \]

\[ K_{p} = -30 \times 10^{-6} \, A/V^2 \]

\[ V_{p} = -0.4 \, V \]

\[ G_{ff} = 0.25 \, V \]

\[ W/L = 9/2 \]

\[ C_{gd} = 0.3 \, \text{ff} \]

\[ C_{gd} = 0.6 \, \text{ff} \]

\[ T_{PLH} = 31.8 \, \text{ps} \]

\[ T_{PHL} = 39.5 \, \text{ps} \]

\[ R_{eq} = 8.68 \, \text{k\Omega} \]

\[ R_{eq} = 6.99 \, \text{k\Omega} \]

\[ C_{L} = 6.6 \, \text{ff} \]

\[ C_{L} = 7.2 \, \text{ff} \]

\[ \text{SPICE: } T_{PLH} = 37.5 \, \text{ps} \quad (C_{gd} = 0.3 \, \text{ff}) \]

\[ T_{PLH} = 41.8 \, \text{ps} \quad (C_{gd} = 0.6 \, \text{ff}) \]

b) Due to capacitive coupling, we see that the output does not behave exactly as expected:

\[ V_{in} \]

\[ V_{out} \]

\[ t \]

To estimate the magnitude of this overshoot, we can analyze this circuit:

\[ U_{in} \]

\[ U_{out} \]

\[ 2.5 \, \text{V} \]

\[ 2.5 \, \text{V} \]
Initially, the output is connected to the supply through the PMOS device ($R_p$). If we then step the input from 0–0.25V, assuming that neither transistor has had time to react, we will have charge redistribution at the output nodes:

$$
\begin{align*}
\frac{2.5}{C_i} & \left[ \begin{array}{c}
-Q_1 + \Delta Q \\
-Q_1 - \Delta Q
\end{array} \right] \\
\frac{1}{C_2} & \left[ \begin{array}{c}
-Q_1 + \Delta Q \\
-Q_1 - \Delta Q
\end{array} \right]
\end{align*}
$$

dotted since the PMOS transistor is out of the picture

$\Delta Q$ charge flows from $C_1 \rightarrow C_2$

So we have, more simply:

$$
\begin{align*}
V \left[ \begin{array}{c}
+V_1 - \Delta Q \\
+V_2 - \Delta Q
\end{array} \right] \\
2.5 \left[ \begin{array}{c}
V_1 \\
V_2
\end{array} \right]
\end{align*}
$$

$$
\frac{V_2}{V} = \frac{V_1}{V} = \frac{\Delta Q}{C_2} + \frac{\Delta Q}{C_1}
$$

$$
\frac{1}{C_1} = \frac{1}{C_2} \cdot \frac{C_i C_r}{C_i + C_r} = \frac{C_1}{C_i + C_r}
$$

Therefore we predict the following magnitudes for overshoot:

$$
\begin{align*}
C_{op} = 0.3 \text{ff} \quad 2.5 \left( \frac{0.3 \text{ff}}{0.3 \text{ff} + 0.6 \text{ff}} \right) &= 0.12 \text{V} \\
C_{op} = 0.6 \text{ff} \quad 2.5 \left( \frac{0.6 \text{ff}}{0.6 \text{ff} + 0.6 \text{ff}} \right) &= 0.23 \text{V}
\end{align*}
$$

Note that everything is symmetrical so the formula holds for the low to high transition as well.

SPICE gives overshoot as 0.18V ($C_{op} = 0.3 \text{ff}$) and 0.29V ($C_{op} = 0.6 \text{ff}$).