Problem 1 – Generating a Voltage Transfer Characteristic

1A Draw the VTC for this circuit. Determine (or estimate, if necessary, from your VTC) the following parameters: $V_{OH}$, $V_{OL}$, $V_M$

We are given both load line plots for the active PMOS device and the non-linear device of the shaded box. How do we link the information provided by these curves to generate information about input and output voltages? First, we need to realize that the output voltage of the PMOS device determines what the voltage drop across the shaded box. That is,

$$V_{DD} = -V_{DS} + V_{\text{shaded-box}}$$

$$V_{out} = V_{DD} + V_{DS}$$

$$V_{in} = V_{DD} + V_{GS}$$

Since we know I-V relationships in each device AND also the fact that the current through one must be equal to the current through the other, we can manipulate the curves to tell us something. Using the relation above, we can superimpose a horizontally-translated version of the shaded-box’s I-V curve onto that of the PMOS’ curves. The intersections are the operating points of this circuit and will give us the input-output voltage relationships we need to build our VTC. Below is the revised, superimposed graph with the intersections labeled:
The resolution of our plot will not be as fine as we’d like, but you can see how if we had more points, the curve becomes more and more accurate. You may plot your VTC whichever way you like, but I chose to use Excel. You may even do it by hand! I also plotted Vin=Vout (dotted line).

Looking at the VTC, it is quite easy to determine what $V_{OH}$, $V_{OL}$, and $V_M$ are:

$V_{OH} \approx 1.8 \text{V}$ -- When the PMOS transistor turns on, it tries to pull Vout high. At the same time, the shaded-box is trying to pull Vout low. Depending on the ratio of their effective resistances, $V_{OL}$ will change. This is something that will be discussed later on the semester when Ratioed Logic is covered in the course.

$V_{OL} = 0 \text{V}$ – PMOS off, shaded-box offers resistive path to bring Vout low, assuming the output is driving a capacitive load or infinite resistance (which is often true for analysis questions like these)

$V_M \approx 0.8 \text{V}$ – Not the 1.25V we were hoping to get in an ideal symmetrical VTC for an inverter.

**1B** This circuit can be used as an alternative to a traditional CMOS inverter (where the non-linear device is an NMOS transistor). From the concepts discussed thus far in lecture and from the results of your VTC, what are the disadvantages of this method?
Incomplete pull-up of the output node means we don’t get full rail-to-rail swing at the output. This also means that we have static power consumption because there is always a direct path from supply to ground when the PMOS transistor is on. For low power applications, we aim to minimize static power.

The VTC also is asymmetric, meaning that a rising transition and its associated output have different large signal and timing characteristics as a falling transition. Uneven noise margins is also another problem.

**Problem 2 – Analysis Using the Unified Model**

2A  From the figure above, determine the following parameters: \( V_{T0}, \lambda, \gamma \).

\( V_{T0} \)  This one should immediately signal you to look at a curve(s) that don’t have body-effect. That means \( V_{BS} = 0V \). Pick two points, each from different curves that satisfy the no-body-effect condition. Make sure they’re in the same operating region too!

<table>
<thead>
<tr>
<th>Point</th>
<th>( V_{GS} )</th>
<th>( V_{DS} )</th>
<th>( I_D )</th>
<th>Operating Region</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2.5V</td>
<td>1.8V</td>
<td>300uA</td>
<td>saturation</td>
</tr>
<tr>
<td>B</td>
<td>2.0V</td>
<td>1.8V</td>
<td>160uA</td>
<td>saturation</td>
</tr>
</tbody>
</table>

The reason why I chose points with the same \( V_{DS} \) will be evident once I work through the math.

\[
I_{D,A} = \frac{1}{2} k_p \left( \frac{W}{L} \right) V_{GS,A} - V_{T0}^2 (1 + \lambda \cdot V_{DS,A}) \]

\[
I_{D,B} = \frac{1}{2} k_p \left( \frac{W}{L} \right) V_{GS,B} - V_{T0}^2 (1 + \lambda \cdot V_{DS,B}) \]

\[
\frac{300}{160} = \frac{(2.5 - V_{T0})^2}{(2.0 - V_{T0})^2} \]

\( V_{T0} = 0.64V \)

As you can see, in order for me to isolate \( V_{T0} \), I needed to make sure I can cancel as many variables to be able to solve the equation.

\( \lambda \)  We can use the same methodology as above. This time, we want to keep \( V_{GS} \) constant.

<table>
<thead>
<tr>
<th>Point</th>
<th>( V_{GS} )</th>
<th>( V_{DS} )</th>
<th>( I_D )</th>
<th>Operating Region</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2.5V</td>
<td>2.4V</td>
<td>310uA</td>
<td>saturation</td>
</tr>
<tr>
<td>B</td>
<td>2.5V</td>
<td>1.8V</td>
<td>300uA</td>
<td>saturation</td>
</tr>
</tbody>
</table>

\[
I_{D,A} = \frac{1}{2} k_p \left( \frac{W}{L} \right) V_{GS,A} - V_T^2 (1 + \lambda \cdot V_{DS,A}) \]

\[
I_{D,B} = \frac{1}{2} k_p \left( \frac{W}{L} \right) V_{GS,B} - V_T^2 (1 + \lambda \cdot V_{DS,B}) \]

\[
\frac{310}{300} = \frac{(1 + \lambda \cdot 2.4)}{(1 + \lambda \cdot 1.8)} \]

\( \lambda = 0.0617V^{-1} \)
It shouldn’t be a surprise, but that leaves us to keep almost everything constant except for $V_{SB}$.

<table>
<thead>
<tr>
<th>Point</th>
<th>$V_{SB}$</th>
<th>$V_{GS}$</th>
<th>$V_{DS}$</th>
<th>$I_D$</th>
<th>Operating Region</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.0V</td>
<td>2.0V</td>
<td>1.2V</td>
<td>105uA</td>
<td>saturation</td>
</tr>
<tr>
<td>B</td>
<td>0.0V</td>
<td>2.0V</td>
<td>1.2V</td>
<td>150uA</td>
<td>saturation</td>
</tr>
</tbody>
</table>

$$I_{D,A} = \frac{1}{2} k_p \left( \frac{W}{L} \right) \left( V_{GS,A} - V_T \right)^2 (1 + \lambda \cdot V_{DS,A})$$

$$I_{D,B} = \frac{1}{2} k_p \left( \frac{W}{L} \right) \left( V_{GS,B} - V_{T0} \right)^2 (1 + \lambda \cdot V_{DS,B})$$

$$\frac{105}{150} = \frac{(2.0 - V_T)^2}{(2.0 - 0.64)^2}$$

$$V_T = 0.862 \text{V}$$

Now solve for $\gamma$ using the following equation:

$$V_T - V_{T0} = \gamma \left( \sqrt{\left| V_{SB} - 2V_F \right|} - \sqrt{2V_F} \right)$$

$$0.862 - 0.64 = \gamma \left( \sqrt{0.6} - \sqrt{0.6} \right)$$

$$\gamma = 0.453 V^{1/2}$$

### 3 Using SPICE, generate the family of curves for an NMOS transistor with the following parameters.

The following is a listing of my example SPICE deck. Yours may be different, but the curves should be the same. Not many of you may be familiar with the .ALTER statement, which allows me to change parameters and resimulate automatically. In addition, your plots may be the opposite in magnitude from my curves because SPICE has a weird convention of current directions. As long as the absolute magnitude of your numbers are correct, you should be in good shape!

```
PS1 2B NMOS Curves
.lib '/home/ff/ee141/MODELS/g25.mod' TT
.param pos=0
.param supply=2.5

* m1 d g s b pmos w=2.0u l=0.25u
m1 d g s b nmos w=2.0u l=0.25u
vgs g s 0
vds d s 0
vsb s b pos
vb b 0 0

* .dc vds -2.5 0 .1 vgs -2.3 -0.7 0.4
 .dc vds 0 2.5 .1 vgs 0.7 2.3 0.4
.plot LX4(M1)
.option post=2 nomod
.alter
.param pos=0.5
.alter
.param pos=1.2
.end
```
The plots are below:

Dotted purple lines are for V_{sb} = 0 V. Increasing V_{gs} from bottom to top.
Dotted red lines are for V_{sb} = 0.5 V. Increasing V_{gs} from bottom to top.
Solid black lines are for V_{sb} = 1.2 V. Increasing V_{gs} from bottom to top.

**Problem 4 – Device Parameters Part 2**

<table>
<thead>
<tr>
<th>Measurement Number</th>
<th>V_{GS}</th>
<th>V_{DS}</th>
<th>V_{SB}</th>
<th>I_D</th>
<th>Operation Region</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-2.5V</td>
<td>-2.5V</td>
<td>0</td>
<td>-84.375uA</td>
<td>velocity saturation</td>
</tr>
<tr>
<td>2</td>
<td>1.0V</td>
<td>1V</td>
<td>0</td>
<td>0.0</td>
<td>cutoff</td>
</tr>
<tr>
<td>3</td>
<td>-0.7V</td>
<td>-0.8V</td>
<td>0</td>
<td>-1.04uA</td>
<td>saturation</td>
</tr>
<tr>
<td>4</td>
<td>-2.0V</td>
<td>-2.5V</td>
<td>0</td>
<td>-56.25uA</td>
<td>velocity saturation</td>
</tr>
<tr>
<td>5</td>
<td>-2.5V</td>
<td>-2.5V</td>
<td>-0.8V</td>
<td>-72.0uA</td>
<td>velocity saturation</td>
</tr>
<tr>
<td>6</td>
<td>-2.5V</td>
<td>-1.5V</td>
<td>0</td>
<td>-80.625uA</td>
<td>velocity saturation</td>
</tr>
<tr>
<td>7</td>
<td>-2.5V</td>
<td>-0.8V</td>
<td>0</td>
<td>-66.56uA</td>
<td>linear</td>
</tr>
</tbody>
</table>

**4A** Is the measured transistor a PMOS or an NMOS device? Explain your answer.

This is a PMOS device. Negative gate-source, drain-source, currents should be your biggest hint.

**4B** From measurements above, determine the following parameters: \( V_{TO}, \gamma, \lambda. \)
This problem is solved using the EXACT same method as problem 2, except that the points are already chosen for you. I will skip the equations and state the answers.

\[ V_{T0} = -0.5V \] – The device is in velocity saturation, so we use the velocity saturation equations of the unified model. The measurements you should use are 1 and 4.

\[ \gamma = -0.538V^{1/2} \] – Use measurements 1 and 5 in the same fashion as problem 2A. Use the **VELOCITY SATURATION** equations from the unified model.

\[ \lambda = -0.05V^{-1} \] – Use measurements 1 and 6 in the same fashion as problem 2A. Use the **VELOCITY SATURATION** equations from the unified model.

4C Complete the missing column in the table above using the values you obtained in 3B. Fill in either “LINEAR”, “CUTOFF”, “SATURATION”, or “VEL. SATURATION.” <You don’t have to recopy the whole table, just the last column is sufficient.>

The answers are provided in the table above and are shaded in the last column.