Why Modeling?

- Analog circuits more sensitive to detailed transistor behavior
  - Precise currents, voltages, etc. matter
  - Digital circuits have much larger “margin of error”

- Models allow us to reason about circuits
  - Provide window into the physical device and process
  - “Experiments” with SPICE much easier to do
Levels of Abstraction

- Best abstraction depends on questions you want to answer

- Digital functionality:
  - MOSFET is a switch

- Digital performance:
  - MOSFET is a current source and a switch

- Analog characteristics:
  - MOSFET described by BSIM with 400 parameters?
  - MOSFET described by measurement results?

Why not Square Law?

- Square law model most widely known:
  \[
  I_{D,sat} = \frac{1}{2} \cdot \mu_n \cdot C_{ox} \cdot \frac{W}{L} \cdot (V_{GS} - V_{th})^2
  \]

- But, totally inadequate for “short-channel” behavior

- Also doesn’t capture moderate inversion
  - (i.e., in between sub-threshold and strong inversion)
Square Law Model Assumptions

- Charge density determined only by vertical field
- Drift velocity set only by lateral field
- Neglect diffusion currents (“magic” $V_{th}$)
- Constant mobility
- And many more…

A Real Transistor

- **Gate Electrode**
  - Gate Depletion
  - Quantum Effect

- **S/D Engineering**
  - S/D resistances
  - S/D leakage

- **Retrograde Doping**
  - Body effect

- **Ultra-thin Gate Dielectric**
  - Direct Tunneling Current
  - Quantum Effects

- **Short Channel Effects**
  - Velocity Saturation and Overshoot
  - Source-end Velocity Limit

- **Pocket Implant**
  - Reverse short channel effect
  - Slower output resistance scaling with L
Now What?

- Rely purely on simulator to tell us how devices behave?
  - Models not always based on real measurements
  - Model extraction is hard
  - Models inherently compromise accuracy for speed

- Need to know about important effects
  - So that know what to look for
  - Model might be wrong, or doesn’t automatically include some effects
    - E.g., gate leakage

Output Resistance: CLM

- “Channel Length Modulation”
  - Depletion region varies with $V_{ds}$
  - Changes effective channel length

- If perturbation is small:
  \[
  I \propto \frac{1}{L - \delta L(V_{ds})} \approx \frac{1}{L} \left( 1 + \frac{\delta L(V_{ds})}{L} \right) \quad \frac{I_d}{I_{d_0}} = (1 + \lambda V_{ds})
  \]
Output Resistance: DIBL

• “Drain Induced Barrier Lowering”
  - Drain controls the channel too
    - Charge gets imaged – lowers effective $V_{th}$
    - Model with $V_{th} = V_{th0} - \eta V_{DS}$

Output Resistance: SCBE

• “Substrate Current Body Effect”
• At high electric fields, get “hot” electrons
  - Have enough energy to knock electrons off Si lattice (impact ionization)

• Extra $e^-$ - $h^+$ pairs – extra (substrate) current
  - Models usually empirical
  $$I_{sub} = \frac{A_i}{B_i} I_{ds} (V_{ds} - V_{dsat}) \exp \left( - \frac{B_i l}{V_{ds} - V_{dsat}} \right)$$
Output Resistance Mechanisms

- All effects active simultaneously
- CLM at relatively low fields
- DIBL dominates for high fields
- SCBE at very high fields

Velocity Saturation

- Drift velocity initially increases linearly with field
- Eventually carriers hit a “speed limit”
- In the limit, \( I_D \propto (V_{GS} - V_{th}) \)
Vertical Field Mobility Reduction

- Mobility actually depends on gate field
  - “Hard to run when there is wind blowing you sideways (into a wall)”

- More technical explanation:
  - E-field pushes carriers close to the surface
  - Enhanced scattering lowers mobility

\[
\mu = \frac{\mu_0}{1 + \theta(V_{GS} - V_T) + \theta_B V_{SB}}
\]
Reverse Short-Channel Effect

Sub-Threshold Region

- Current doesn’t really go to 0 at $V_{GS} = V_{th}$
- Lateral BJT:
Weak Inversion Channel Potential

• “Base” controlled through capacitive divider

\[ \delta V_{ch} \approx \frac{C_{ox}}{C_{dep} + C_{ox}} \delta V_g = \frac{\delta V_g}{n} \]

• Non-ideality factor of channel control \( n > 1 \):

\[ n = 1 + \frac{C_{dep}}{C_{ox}} = 1 + \frac{\epsilon_{dep} \epsilon_{ox}}{\epsilon_{ox} \epsilon_{dep}} \]

• (\( n \) varies somewhat with bias – const. approx. usually OK)

Weak Inversion Current

• Current set by diffusion – borrow BJT equation:

\[ I_{ds} = \frac{W}{L} I_{ds,0} e^{\frac{q(V_{gs} - V_T)}{n k T}} \left( 1 - e^{-\frac{q V_{ds}}{k T}} \right) \]
Operating in Weak Inversion

- Usually considered “slow”:
  - “large” $C_{GS}$ for “little” current drive (see later)

- But, weak (or moderate) inversion becoming more common:
  - Low power
  - Submicron $L$ means “high speed” even in weak inversion

- Not well modeled, matching poor:
  - $V_{TH}$ mismatch amplified exponentially
  - Avoid in mirrors

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Moderate Inversion

- Moderate inversion: both drift and diffusion contribute to the current.

- Closed form equations for this region don’t really exist.
Patching Models?

• Have “good” models for weak inversion and strong inversion.
  • Why not just interpolate in between?

• Example (EKV):

\[
I_{DS} = \frac{W}{L} \mu C_{ox}(2n) \left( \frac{kT}{q} \right)^2 \left( \ln \left( 1 + e^{\frac{V_{GS} - V_{TH} - V_{IL}}{nV_{IL}}} \right) \right)^2 \left( \ln \left( 1 + e^{\frac{V_{DS} - V_{IL}}{2nV_{IL}}} \right) \right)^2
\]

BSIM

• *Berkeley Short-channel IGFET Model (BSIM)*
  • Industry standard model for modern devices
  • BSIM3v3 is model for this course

• Typically 40-100 parameters
  • Advanced software and expertise needed to perform extraction
BSIM “Hand Calculation” Model

- Requires many, many, many… assumptions

- Vertical mobility degradation:

  Define: \( u_d = \frac{UA}{I_{ds}} \) mobility degradation coefficient

  \[ u_d \approx 0.5 \text{V}^{-1} \text{ for } t = 10 \text{nm} \]

- Velocity saturation:

  Define: \( E_c = \frac{2V_m}{U/0} \) critical \( E \)-field for velocity saturation

  \[ E_c \approx 2 \times 10^4 \text{V/cm} \text{ (typical value)} \]

---

Strong Inversion Current

\[
V_{DS} = (V_G - V_T) \left[ \frac{1 + u_d(V_G - V_T)}{1 + u_d(V_G - V_T) + \frac{V_m}{E_c L}} \right]
\]

\[
I_{DS} = \mu C_{ov} \frac{W}{2L} \left[ \frac{(V_G - V_T)^2}{1 + u_d(V_G - V_T) + \frac{V_m}{E_c L}} \right] = I_{DS(long)} \left[ \frac{1}{1 + u_d(V_G - V_T) + \frac{V_m}{E_c L}} \right]
\]
Equations of Derivatives

\[ g_{out} = \frac{I_{Dsat}}{(V_G - V_T)} \left[ 1 + \frac{I_{Dsat}}{I_{Dsat(long)}} \right] = \frac{I_{Dsat}}{(V_G - V_T)} \left[ 1 + \frac{1}{1 + \left( u_d + \frac{1}{E_c} \right)(V_G - V_T)} \right] \]

\[ r_{out} = \frac{2[(V_D - V_{Dsat}) + [1 + u_d(V_G - V_T)](V_G - V_T)]L^2}{\mu_C \cdot W \cdot P_{CLM} \cdot [1 + u_d(V_G - V_T)]^2} \]

\[ = \frac{[V_D - V_{Dsat}) + [1 + u_d(V_G - V_T)](V_G - V_T)]L}{I_{Dsat(long)} \cdot P_{CLM} \cdot [1 + u_d(V_G - V_T)]} \]

with \( l = \sqrt{\frac{3e_{ox}}{\epsilon}} \)

- Required parameters: \( W, L, TOX, U_0, UA, VSAT, VTH0, PCLM, XJ \)

Fitting Results

Comparison between full and simplified model

Parameter detail: TSMC 0.18\(\mu\)m process
- \( t_{ox} \): 4.1nm, \( H \): 10\(\mu\)m, \( V_{TH} \): 0.39V
Weakness of Model First Derivatives

```
VG (V)

gm (mA/V)

Full BSIM3
Hand calculation
VD = 1.8V
VD = 0.1V

Rout (kΩ)

Full BSIM3
Hand calculation
VD = 2.0V
VD = 1.8V

“Hand Model” Conclusion

- Even “simple” model is not convenient
  - rₒ is key for gain, but really hard to model
  - Missing important regions such as moderate inversion

- Hand models really best to build intuition

- But for design (i.e., how to choose W, L, etc.):
  - Will learn how to use the simulator as a “calculator”
```