Why Modeling?

• Analog circuits more sensitive to detailed transistor behavior
  - Precise currents, voltages, etc. matter
  - Digital circuits have much larger “margin of error”

• Models allow us to reason about circuits
  - Provide window into the physical device and process
  - “Experiments” with SPICE much easier to do

Why not Square Law?

• Square law model most widely known:
  \[ I_{D,IS} = \frac{1}{2} \mu C_w \frac{W}{L} (V_{GS} - V_T)^2 \]

• But, totally inadequate for “short-channel” behavior

• Also doesn’t capture moderate inversion
  - (i.e., in between sub-threshold and strong inversion)

Levels of Abstraction

• Best abstraction depends on questions you want to answer

  • Digital functionality:
    - MOSFET is a switch
  
  • Digital performance:
    - MOSFET is a current source and a switch

  • Analog characteristics:
    - MOSFET described by BSIM with 400 parameters?
    - MOSFET described by measurement results?

Square Law Model Assumptions

• Charge density determined only by vertical field

• Drift velocity set only by lateral field

• Neglect diffusion currents (“magic” \( V_{th} \))

• Constant mobility

• And many more…

A Real Transistor

- Ultra-thin Gate Dielectric
- Direct Tunneling Current
- Quantum Effects

- S/D Engineering
- S/D resistances
- S/D leakage

- Short Channel Effects
- Velocity Saturation and Overshoot
- Source-end Velocity Limit

- Retrograde Doping
- Body effect

- Pocket Implant
- Reverse short-channel effect
- Slower output resistance scaling with L
Now What?
• Rely purely on simulator to tell us how devices behave?
  • Models not always based on real measurements
  • Model extraction is hard
  • Models inherently compromise accuracy for speed

• Need to know about important effects
  • So that know what to look for
  • Model might be wrong, or doesn’t automatically include some effects
    • E.g., gate leakage

Output Resistance: CLM
• “Channel Length Modulation”
  • Depletion region varies with $V_{DS}$
  • Changes effective channel length

  • If perturbation is small:
    \[ I \propto \frac{1}{L} - \frac{1}{L} \left( 1 + \frac{\delta L(V_{DS})}{L} \right) \]
    \[ \frac{I_{DS}}{I_{D1}} = (1 + \lambda V_{DS}) \]

Output Resistance: DIBL
• “Drain Induced Barrier Lowering”
  • Drain controls the channel too
    • Charge gets imaged – lowers effective $V_{in}$
    • Model with $V_{in} = V_{th} - \eta V_{DS}$

Output Resistance: SCBE
• “Substrate Current Body Effect”
  • At high electric fields, get “hot” electrons
    • Have enough energy to knock electrons off Si lattice (impact ionization)

  • Extra $e^-$ - $h^+$ pairs – extra (substrate) current
    • Models usually empirical
    \[ I_{sub} = A_1 I_{DS}(V_{DS}) V_{DS}^{0.07} \left( \frac{B_{HL}}{V_{DS} - V_{th}} \right) \]

Output Resistance Mechanisms
• All effects active simultaneously
  • CLM at relatively low fields
  • DIBL dominates for high fields
  • SCBE at very high fields

Velocity Saturation
• Drift velocity initially increases linearly with field
  • Eventually carriers hit a “speed limit”
  • In the limit, $I_0 \propto (V_{GS} - V_{th})$
Vertical Field Mobility Reduction

- Mobility actually depends on gate field
  - “Hard to run when there is wind blowing you sideways (into a wall)”
- More technical explanation:
  - E-field pushes carriers close to the surface
  - Enhanced scattering lowers mobility

\[ \mu = \frac{\mu_0}{1 + \theta(V_{GS} - V_T) + \theta RV_{SB}} \]

Sub-Threshold Region

- Current doesn’t really go to 0 at \( V_{GS} = V_{th} \)
- Lateral BJT:

Halo Doping

- “Base” controlled through capacitive divider

\[ \delta V_{CB} \approx \frac{C_{ox}}{C_{dep} + C_{ox}} \delta V_T = \frac{\delta V_T}{n} \]
- Non-ideality factor of channel control \( n > 1 \):

\[ n = 1 + \frac{C_{dep}}{C_{ox}} = 1 + \frac{C_{dep} \delta x}{C_{ox} \delta x_{dep}} \]
- \( (n \) varies somewhat with bias – const. approx. usually OK)

Reverse Short-Channel Effect

- Current set by diffusion – borrow BJT equation:

\[ I_{ds} = \frac{W}{L} I_{ds0} e^{\frac{(V_{GS}-V_{TP})}{\alpha kT}} \left( 1 - e^{-\frac{\eta V_{DS}}{kT}} \right) \]
Operating in Weak Inversion

- Usually considered “slow”:
  - “large” $C_{ox}$ for “little” current drive (see later)
- But, weak (or moderate) inversion becoming more common:
  - Low power
  - Submicron L means “high speed” even in weak inversion
- Not well modeled, matching poor:
  - $V_{th}$ mismatch amplified exponentially
  - Avoid in mirrors

Moderate Inversion

- Moderate inversion: both drift and diffusion contribute to the current.
- Closed form equations for this region don’t really exist.

BSIM

- Berkeley Short-channel IGFET Model (BSIM)
  - Industry standard model for modern devices
  - BSIM3v3 is model for this course
- Typically 40-100 parameters
  - Advanced software and expertise needed to perform extraction

BSIM “Hand Calculation” Model

- Requires many, many, many... assumptions

- Vertical mobility degradation:
  \[
  u_m = \frac{u_m}{u_m} \text{ mobility degradation coefficient}
  \]
  \[
  u_m = 0.5\mu \text{ mV}^{-1} \text{ for } L = 10\mu m
  \]

- Velocity saturation:
  \[
  E_s = \frac{2u_m}{W/2L} \text{ critical } E\text{-field for velocity saturation}
  \]
  \[
  E_s = 2 \times 10^6 \text{ V/cm} \text{ (typical value)}
  \]

Patching Models?

- Have “good” models for weak inversion and strong inversion.
  - Why not just interpolate in between?

Example (EKV):

\[
I_{DR} = \frac{W}{2L} \left[ \frac{1}{1 + \frac{u_m}{u_m} (V_d - V_F)} \right] \left( \frac{n}{1 + \frac{u_m}{u_m} \frac{E_{sat}}{E}} \right) \left( \frac{1}{1 + \frac{u_m}{u_m} \frac{E_{sat}}{E}} \right)
\]

Strong Inversion Current

\[
I_{inw} = \frac{W}{2L} \left[ \frac{1}{1 + \frac{u_m}{u_m} (V_d - V_F)} \right] \left( \frac{n}{1 + \frac{u_m}{u_m} \frac{E_{sat}}{E}} \right) \left( \frac{1}{1 + \frac{u_m}{u_m} \frac{E_{sat}}{E}} \right)
\]

\[
I_{inw} = \frac{u_m}{2L} \left[ \frac{1}{1 + \frac{u_m}{u_m} \frac{E_{sat}}{E}} \right] \left( \frac{1}{1 + \frac{u_m}{u_m} \frac{E_{sat}}{E}} \right)
\]

\[
I_{inw} = \frac{u_m}{2L} \left[ \frac{1}{1 + \frac{u_m}{u_m} \frac{E_{sat}}{E}} \right] \left( \frac{1}{1 + \frac{u_m}{u_m} \frac{E_{sat}}{E}} \right)
\]
Equations of Derivatives

\[ I_{out} = \frac{I_{sat}}{1 + \left( \frac{V_C - V_t}{V_{th}} \right)^2} \]

\[ R_{on} = \frac{2(V_C - V_{th})}{I_{sat} + |I_{sat}|} \left( \frac{1}{1 + \left( \frac{V_C - V_t}{V_{th}} \right)^2} \right) \]

with \( I = \frac{V_C - V_t}{V_{th}} \)

• Required parameters: \( W, L, TON, U0, U_A, VSAT, VTH0, PCLM, XJ \)

“Hand Model” Conclusion

• Even “simple” model is not convenient
  • \( r_o \) is key for gain, but really hard to model
  • Missing important regions such as moderate inversion

• Hand models really best to build intuition
  • But for design (i.e., how to choose \( W, L, \) etc.):
    • Will learn how to use the simulator as a “calculator”

Fitting Results

Comparison between full and simplified model

Parameter detail: TSMC 0.18\( \mu \)m process

\( r_o : 4.1\Omega, W=10\mu m, V_{th}=0.39V \)

Weakness of Model First Derivatives