Electronic Noise

• Why is noise important?
  • Sets minimum signals we can deal with – often sets lower limit on power

• Signal-to-noise ratio
  • Signal Power $P_{\text{sig}} \sim (V_{DD})^2$
  • Noise Power $P_{\text{noise}} \sim kT/C$
  • SNR = $P_{\text{sig}} / P_{\text{noise}}$

• Technology Scaling
  • $V_{DD}$ goes down → lower signal
  • Increase C to compensate → increases power

Thermal Noise of a Resistor

• Origin: Brownian Motion
  • Thermally agitated particles
  • E.g. ink in water, electrons in a conductor

• Available noise power: $P_n = kT\Delta f$
  • Noise power in bandwidth $\Delta f$ delivered to a matched load
  • Example: $\Delta f = 1\text{Hz} \Rightarrow P_n = 4 \times 10^{-21}\text{W} = -174\text{ dBm}$

Resistor Noise Model

$P_n = kT\Delta f \Rightarrow \sqrt{P_n} = \frac{\sqrt{4kT\Delta f}}{4R}$

Mean square noise voltage:

$V_{\text{rms}}^2 = 4kT\Delta f$
**Thermal Noise**
- Present in all dissipative elements
  - I.e., resistors
- Independent of DC current flow
- Random fluctuations of $v(t)$ or $i(t)$
  - Mean is 0
  - Distribution (pdf) is Gaussian
  - Power spectral density is “white”
    - Up to about $1/\tau = 2 \times 10^6$ GHz
  - $k_B = 4.6 \times 10^{-21}$ J (T = 290K = 16.9°C)
- Example: $R = 1 \Omega \rightarrow 4nV/\sqrt{\text{Hz}}$
  - $1\text{MHz bandwidth} \rightarrow \sigma = 4\mu V$

**Noise of Passive Networks**
- Capacitors, Inductors
- Noise calculations
  - Instantaneous voltages add
  - Power spectral densities add
  - RMS voltages do NOT add
- Example: $R_1 + R_2$ in series
- Generalization to arbitrary RLC networks

**Shot noise**
- Zero mean, Gaussian pdf, white
- Independent of temperature
- $i^2 = 2qI_{dc}\Delta f$
- Example: $I_p = 1\text{mA} \rightarrow 17.9\text{pA/}\sqrt{\text{Hz}}$
  - $1\text{MHz bandwidth} \rightarrow \sigma = 17.9\text{nA}$
- Shot noise versus thermal noise
  - $i_{\text{shot}} = I_p/(k_BT/q)$
  - Thermal noise density: $4k_B T R_{\text{source}} = 4qI_{dc}$
  - Shot noise half of this (current flow in 1 direction)

**BJT Noise**
- Just like diodes: shot noise
  - Collector and base noise partially correlated
- Extrinsic resistors contribute noise
  - Small signal resistors (e.g., $r_e$) don’t
  - These aren’t physical resistors

**FET Noise**
- Channel resistance contributes thermal noise
- Channel conductance:
  - $g_{\text{soa}} = \mu C_{\text{ox}} W / L (V_{\text{ds}} - V_{\text{th}}) = g_{\text{so}}$
- Noise injection is actually distributed across the channel (note $\gamma$):
  - $i^2 = 4kT \gamma g_{\text{mo}} \Delta f$

**More Fundamental Expression**
- More fundamental equation uses channel charge
  - $[\text{Tsividis}]$
  - $i^2 = 4kT W / L^2 (Q_{\text{dc}} + \Delta Q_{\text{dc}}) \Delta f$
  - When $V_{\text{ds}} = 0$, device is truly a resistor:
    - $i^2 = 4kT W / L^2 C_{\text{ox}} (V_{\text{GS}} - V_{\text{th}}) \Delta f$
Strong Inversion Noise

- In saturation, drain current noise is
  \[ \overline{i_d^2} = 4kT \frac{2W}{3L} \mu C_{ox} (V_{GS} - V_T) \Delta f \]

- For long channel model, can substitute \( g_m \) for the above factor.

- In practice, form involving actual inversion charge is more accurate
  - This is what SPICE/BSIM use

Weak Inversion

- Weak inversion: BJT \( \rightarrow \) shot noise.
  - Result should be \( \sim 2 \beta_0 \)

- Get the same result from inversion charge expression:
  \[ Q_i = W \frac{q}{2} + Q_{il} = \frac{L^2}{2e} \gamma q \Delta f \left( 1 + e^{-q V_{DS}/kT} \right) \]
  \[ \overline{i_d^2} = 2q \gamma q \Delta f \left( 1 + e^{-q V_{DS}/kT} \right) \Delta f \]

FET Noise Model

- Model neglects intrinsic gate noise
  - BSIM3 does not directly include \( \alpha \)

1/f Noise

- Flicker noise
  - \( K_{f,NMOS} = 2.0 \times 10^{-29} \text{AF} \)
  - \( K_{f,PMOS} = 3.5 \times 10^{-30} \text{AF} \)
  - Strongly process dependent

- Example: \( I_D = 10 \mu A, L = 1 \mu m, C_{ox} = 5.3 \text{fF/\mu m}^2, f = 1 \text{MHz} \)
  - \( f_0 = 1 \text{Hz} \rightarrow \sigma = 722 \text{pA} \)
  - \( f_0 = 1 \text{year} \rightarrow \sigma = 1083 \text{pA} \)

- 1/f noise corner frequency
  - Definition (MOS)
  - Example:
    - \( V^* = 200 \text{mV}, \gamma = 1 \)
    - NMOS: \( L = 0.35 \mu m \rightarrow f_c = 192 \text{kHz} \)
    - PMOS: \( L = 1.00 \mu m \rightarrow f_c = 24 \text{kHz} \)

Thermal Noise for Short Channels

- Strong inversion \( \rightarrow \) thermal noise
  - Drain current: \( g_{m0} \) is what you really care about
  - \( \overline{i_d^2} = 4kT \gamma q \Delta f = 4kT \gamma g_m \Delta f \)

- \( g_m \) more convenient for input-referred noise
  - For low field (long \( L \)), \( \gamma = 2/3 \) relates \( g_m \) to \( g_{m0} \)
  - For high field, use \( \alpha \) to capture drop in \( g_m \)
  - High-field noise can be 2-3 times larger than low field

- MOS actually has intrinsic gate induced noise (142/242 topic)
- Gate leakage \( \rightarrow \) shot noise

1/f Noise Corner Frequency

- Definition (MOS)
  - \[ \frac{K_f}{f_c} \frac{V^*}{C_{ox}} = 4kT \gamma \frac{1}{C_{ox}} \]
  - \[ f_c = \frac{K_f}{\gamma} \frac{1}{f} \frac{1}{C_{ox}} \frac{4kT \gamma}{C_{ox}} \]
  - \[ f_c = \frac{K_f}{\gamma} \frac{1}{f} \frac{1}{C_{ox}} \frac{4kT \gamma}{C_{ox}} \]

- Example:
  - \( V^* = 200 \text{mV}, \gamma = 1 \)
  - NMOS: \( L = 0.35 \mu m \rightarrow f_c = 192 \text{kHz} \)
  - PMOS: \( L = 1.00 \mu m \rightarrow f_c = 24 \text{kHz} \)
Noise Calculations

- Method:
  1) Create small-signal model
  2) All inputs = 0 (linear superposition)
  3) Pick output $v_o$ or $i_o$
  4) For each noise source $v_x, i_x$
     Calculate $H_x(s) = \frac{v_o(s)}{v_x(s)}$ ($... i_o, i_x$)
     Total noise at output is:

     $$v_{n,x}(f) = \sum_{x=1}^{X} H_x(s) \int_{-\infty}^{+\infty} v_x(f)$$

     simpler notation: $v_{n,x}(f) = S_x(f)$

     Tedious but simple ...

Example: Common Source