Settling

Why interested in settling?
- Oscilloscope: track input waveform without ringing
- ADC (switched-cap amplifier): gain a signal up a precise amount within $T_{\text{sample}}$

Step Response

Two types of settling “errors”:
- Static
  - Finite gain, capacitor mismatch
- Dynamic
  - Takes time to reach final value
Static Error

\[ \frac{V_o}{V_i} = \frac{c}{1 + \frac{1}{F A_w}} \]

\[ F = \frac{C_f}{C_f + C_i + C_v} \]

\[ c = \frac{C_i}{C_f} \]

Example:
- Closed loop gain: \( c = -4 \), \( C_i = 1 \mu F \), \( C_v = 4 \mu F \), \( C_f = 1 \mu F \)
- \( F = 1/6 \) (\( C_i \) hurts!)

Error specification: <0.1%
- \( FA_v > 1000 \)
- \( A_v > 6000 \) over output range

Single Time Constant Linear Settling

For dynamic settling (and for \( T_0 \gg 1 \)), can generally ignore \( r_o \)

\[ \frac{V_o}{V_i} = \frac{1 - s C_f G_m}{1 + \frac{1}{C_f (1 - F K_v)} FG_w} \]

Dynamic Errors

- Many possible dynamic effects that impact settling error:
  - Finite bandwidth
  - Feedforward zero
  - Non-dominant poles
  - Doublets
  - Slewing

- Approximate analysis approach:
  - Decompose each error source, isolate interactions
  - Add all errors together

Time Domain Step Response

Frequency domain:

\[ V_{s,exp} = \frac{1}{1 + s / z} V_{exp} \]

Time domain:

\[ v_{s,exp}(t) = -V_{exp} e^{-\frac{t}{\tau}} \]

\[ \tau = \frac{1}{s / z} \frac{1}{1 + s / p} \frac{1 + s / z}{1 + s / p} \]

Output step:

\[ V_{o,exp} = \frac{1 + s / z}{1 + s / p} V_{exp} \]

\[ = -\frac{1 + s / z}{1 + s / p} V_{exp} e^{-\frac{t}{\tau}} \]

\[ \tau = \frac{1}{s / z} \frac{1}{1 + s / p} \frac{1 + s / z}{1 + s / p} \]

Exponentially decaying error

Applications?
Case 1: $|p/z| << 1$

$$v_{\text{step}}(t) = -V_0 e^{-t/\tau} \left(1 - e^{-t/\tau}\right)$$

Relative settling error:

$$\epsilon = \frac{v(t \to \infty) - v(t = t_s)}{v(t \to \infty)} = e^{-t/\tau} \ln(1 - \epsilon)$$

- Easiest number to remember: $2.3\tau$ per decade
- Example: 1% settling, 4.6ns clock cycle: $\tau = 1\text{ns}$
- $C_{\text{L,eff}}$ usually set by noise – use settling to determine required $g_m$

Case 2: $|p/z|$ not negligible

$$v_{\text{step}}(t) = -V_0 e^{-t/\tau} \left(1 - \left(1 + \frac{p}{z}\right) e^{-t/\tau}\right)$$

Relative settling error:

$$\epsilon = \frac{v(t \to \infty) - v(t = t_s)}{v(t \to \infty)} = \left(1 + \frac{p}{z}\right) e^{-t/\tau} \ln(1 - \epsilon)$$

- Example:
  - $c = 0.25$, $C_i = 1\text{pF}$, $C_a = 250\text{fF}$, $C_i = 250\text{fF}$, $C_v = 1\text{pF}$
  - $F = 0.67$, $C_{\text{L,eff}} = 1.33\text{pF}$
- $\epsilon = 0.1\%$: $t_s$ (no feedforward) = $6.9\tau$
- $t_s$ (with feedforward) = $-\ln(1e^{-3}/(1+0.67*0.75)) = 7.3\tau$

Non-Dominant Pole

- Ignore feed-forward zero for simplicity
  - (Just increases final swing by $1 + FCf/C_{\text{L,eff}}$)

$$H(s) = \frac{v_{\text{out}}}{v_{\text{in}}} = -\frac{1}{1 + \frac{C_{\text{L,eff}}}{C_f}}$$

- Model for non-dominant pole:

$$G_c(s) = \frac{G_{\text{out}}}{1 + s/\omega_c}$$

$$p_x = K\omega_c$$

$\omega_c$ is unity gain bandwidth of $T(s)$

Settling Time

$$t_s(K) \text{ for } \epsilon = 10^{-3}, \tau = 1$$

- Optimum at $K=3.3$
- Avoid $K < 2$
Doublets

- Amplifier model:
  \[ G_m(s) = G_m \left( \frac{1 + s/\omega}{1 + s/\omega_p} \right) \]
  \[ \omega_p = \alpha \omega \quad \text{with} \quad \alpha = 1 + \varepsilon \quad \text{with} \quad |\varepsilon| \ll 1 \]

- Closed-loop gain (ignore feedforward zero):
  \[ \frac{V_o}{V_{in}} = \frac{1}{1 + s/\omega} \left( 1 + \frac{1 + s/\omega}{1 + s/\omega_p} \right) \]
  \[ \omega = \alpha \omega \quad \text{and} \quad \frac{F G_m}{C_{ef}} \]

Doublet Conclusions

- Case A: \( T_d \leq T_i \) i.e. \( \beta \geq 1 \)
  - Doublet settles faster than amplifier
  - Has no impact on overall settling time

- Case B: \( T_d > T_i \)
  - Doublet settles more slowly than amplifier
  - Determines overall settling time (unless \( \varepsilon \) within settling accuracy requirements)

  \[ \Rightarrow \text{Avoid “slow” doublets!} \]

Doublet Analysis

- Step response
  \[ v_{in,exp}(t) = -c V_{in} \left( 1 - A e^{-\alpha \omega} - B e^{-\beta \omega} \right) \]
  \[ B = \varepsilon \frac{\beta}{1 - \beta} \]
  \[ A = 1 - B = 1 \]

Final Note on Doublets

- Transconductor \( \Delta I \) vs. \( \Delta V \):

Slewing

- Model for (nonlinear) slewing amplifier
  - Piecewise linear approximation:
    - Slewing with constant current, followed by
    - Linear settling exponential
      - \( t_s = t_{slow} + t_{s,lin} \)
Slewing Analysis

- Circuit model during slewing:

\[ \begin{align*}
V_x & = \frac{C_s}{C_p + C_L} V_i \\
\text{Vo} & = \text{ISS}
\end{align*} \]

Slewing Analysis (cont.)

- Slewing period:

\[ V_{\text{comp}} = V_{\text{comp}} \left( \frac{C_s}{C_p + C_L} \right) \quad \text{with} \quad C_i = C_s + \frac{C_s C_l}{C_p + C_L} \]

\[ \Delta V = V_{\text{comp}} - V^* \rightarrow \Delta V = \frac{\Delta V}{F} \]

\[ t_{\text{lin}} = \frac{\Delta V}{S R} = \frac{\Delta V}{F_{\text{lin}}} \]

- Linear settling during final \( V^* \) of swing at \( V_x^* \):
- Step during linear settling:

\[ V^* = F \]

- Linear settling time:

\[ t_{\text{lin}} = \ln \left( \frac{e^{V_{\text{lin}}/F}}{V^*} \right) \]