


*EE141-Fall 2012  
Digital Integrated  
Circuits*

Lecture 9  
MOS Transistor Model

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*MOS Transistor*



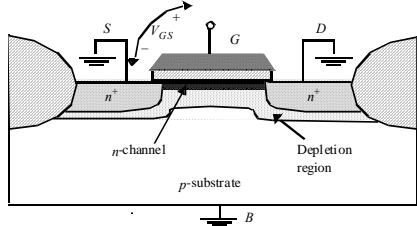
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*Announcements*

- Homework #4 due today
- Homework #5 due next Thurs.
- Midterm #1 Thurs. Oct. 4<sup>th</sup>
  - Location TBA

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*Threshold Voltage: Concept*



- With positive gate bias, electrons pulled toward the gate
- With large enough bias, enough electrons will be pulled to "invert" the surface (p→n type)
- Voltage at which surface inverts: "magic" threshold voltage  $V_T$

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*Class Material*

- Last lecture
  - LE in decoders
- Today's lecture
  - MOS transistor modeling
    - Will see how to use these models to understand tradeoffs between CMOS gate delay, power, etc.
- Reading (3.3.1-3.3.2)

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*The Threshold Voltage*

- Threshold
 
$$V_T = \phi_{FB} + 2\phi_F + \frac{Q_B}{C_{ox}}$$

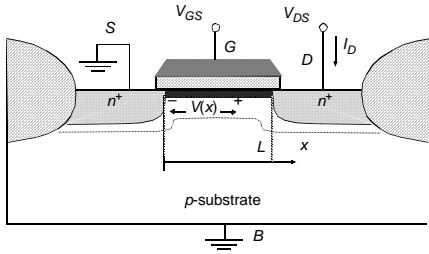
← Depletion charge

$$V_T = V_{T0} + \gamma \cdot (\sqrt{|2\phi_F + V_{SB}|} - \sqrt{2\phi_F})$$
- Fermi potential
 
$$\phi_F = \phi_T \cdot \ln \frac{NA}{n_i}$$

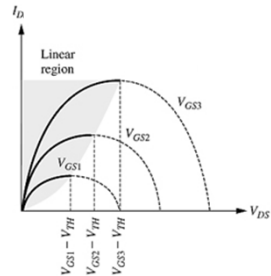
$2\phi_F$  is approximately 0.6V for p-type substrates  
 $\gamma$  is the body factor  
 $V_{T0}$  is approximately 0.45V for a 0.25um process

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*Transistor with Gate and Drain Bias*



*Plot of I-V Curve*



□ Is this really what happens?

*The Drain Current*

□ Charge density:

$$Q_c(x) =$$

□ Velocity:

$$v_n(x) =$$

□ Current:

$$I_D =$$

*Cause of the Problem*

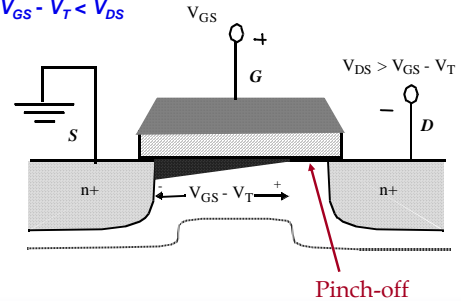
- Why does the current bend down?
- When  $(V_{GS} - V_{TH}) - V_{DS}$  is negative, in our analysis the sign of the carriers changes
  - But transistors don't actually behave this way
- Look at what really happens to channel charge:

*Solving the Drain Current*

□ Integrate along the channel:

*Transistor in Saturation*

$$0 < V_{GS} - V_T < V_{DS}$$



### Saturation

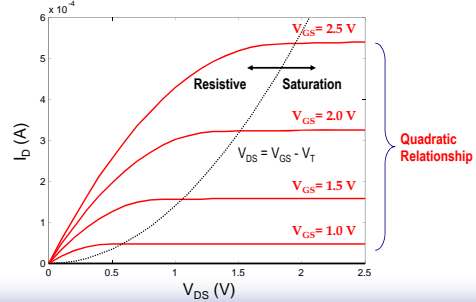
- For  $(V_{GS} - V_T) < V_{DS}$ , the effective drain voltage and current saturate:

$$V_{DS,eff} = (V_{GS} - V_T) \quad I_D = \frac{k'_n}{2} \cdot \frac{W}{L} \cdot (V_{GS} - V_T)^2$$

- Of course, real drain current isn't totally independent of  $V_{DS}$ 
  - For example, approx. for channel-length modulation:

$$I_D = \frac{k'_n}{2} \cdot \frac{W}{L} \cdot (V_{GS} - V_T)^2 \cdot (1 + \lambda \cdot V_{DS})$$

### Current-Voltage Relations: A Good Ol' Transistor



### Modes of Operation

Cutoff:

$$V_{GS} - V_T < 0 \quad I_D = 0$$

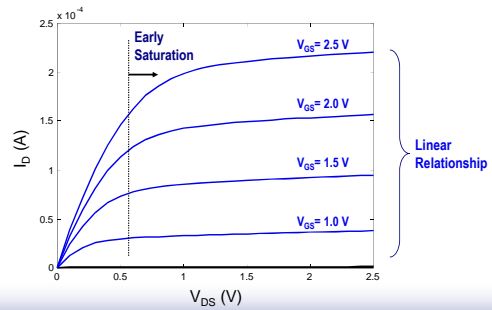
Linear (Resistive):

$$V_{GS} - V_T > V_{DS} \quad I_D = k'_n \cdot \frac{W}{L} \cdot \left[ (V_{GS} - V_T) \cdot V_{DS} - \frac{V_{DS}^2}{2} \right]$$

Saturation:

$$0 < V_{GS} - V_T < V_{DS} \quad I_D = \frac{k'_n}{2} \cdot \frac{W}{L} \cdot (V_{GS} - V_T)^2 \cdot (1 + \lambda \cdot V_{DS})$$

### Current-Voltage Relations: The Deep Sub-Micron Transistor



### Another Way of Looking at This

Cutoff:

$$V_{GS} - V_T < 0 \quad I_D = 0$$

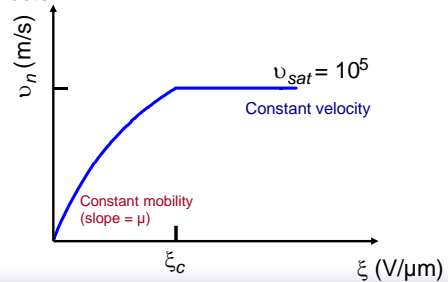
On: (Linear or Saturated)

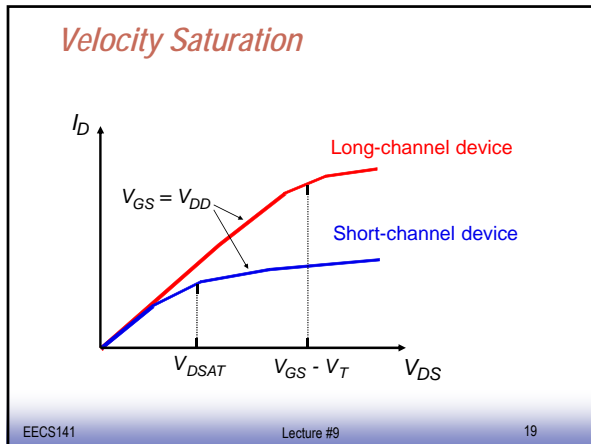
$$V_{DS,eff} = \min(V_{GS} - V_T, V_{DS})$$

$$I_D = k'_n \cdot \frac{W}{L} \cdot \left[ (V_{GS} - V_T) \cdot V_{DS,eff} - \frac{V_{DS,eff}^2}{2} \right]$$

### Velocity Saturation

- Velocity saturates due to carrier scattering effects



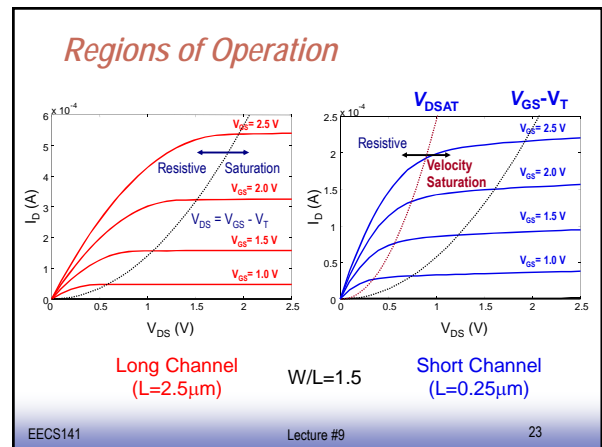
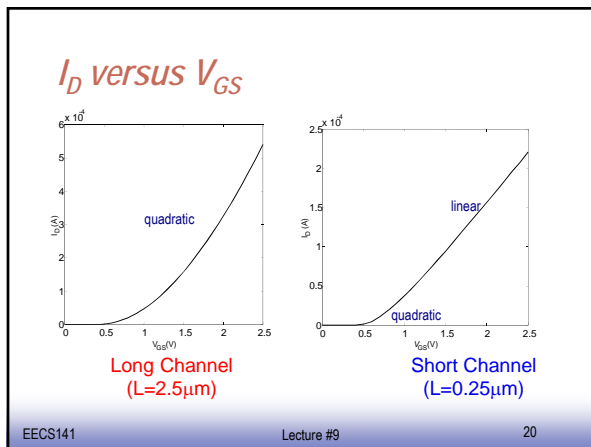


### Velocity Saturation Drain Current

- Saturation occurs when carriers reach  $v_{sat}$ 

$$I_D = WC_{ox}(V_{GS} - V_T - V_{DSAT})v_{sat}$$
- We also know that:
$$I_D = \frac{\mu_n C_{ox}}{1 + (V_{DSAT}/\xi_c)L} \left(\frac{W}{L}\right) \left[ (V_{GS} - V_T)V_{DSAT} - \frac{V_{DSAT}^2}{2} \right]$$
- Equating the two expressions gives  $V_{DSAT}$  and  $I_D$ :
$$V_{DSAT} = \frac{(V_{GS} - V_T)\xi_c L}{(V_{GS} - V_T) + \xi_c L} \quad I_D = Wv_{sat}C_{ox} \frac{(V_{GS} - V_T)^2}{(V_{GS} - V_T) + \xi_c L}$$

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### Including Velocity Saturation

Approximate velocity:

$$v = \frac{\mu_n \xi}{1 + \xi/\xi_c} \text{ for } \xi \leq \xi_c$$

$$= v_{sat} \text{ for } \xi \geq \xi_c$$

Continuity requires that:

$$\xi_c = 2v_{sat}/\mu_n$$

Integrating to find the current again:

$$I_D = \frac{\mu_n C_{ox}}{1 + (V_{DS}/\xi_c)L} \left(\frac{W}{L}\right) \left[ (V_{GS} - V_T)V_{DS} - \frac{V_{DS}^2}{2} \right]$$

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### Models, Models, Models...

- Exact behavior of transistor in velocity sat. somewhat challenging if want simple/easy to use models
- So, many different models developed over the years
  - v-sat, alpha, unified,  $V_T^*$ , etc.
- I often use v-sat model I just presented
  - Works well for calculating LE of complex gates (more later)
  - But still somewhat complicated – often want even simpler model

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### Simplified Velocity Saturation

- Assume velocity perfectly linear until hit  $v_{sat}$

$v_{sat} = 10^5$   
Constant velocity

$\xi_c/2 = v_{sat}/\mu$

$\xi$  (V/ $\mu$ m)

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### A Unified Model for Manual Analysis

define  $V_{GT} = V_{GS} - V_T$

for  $V_{GT} \leq 0$ :  $I_D = 0$

for  $V_{GT} \geq 0$ :

$$I_D = k' \cdot \frac{W}{L} \cdot \left( V_{GT} \cdot V_{DS,eff} - \frac{V_{DS,eff}^2}{2} \right) \cdot (1 + \lambda \cdot V_{DS})$$

with  $V_{DS,eff} = \min(V_{GT}, V_{DS}, V_{D,VSAT})$

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### Simplified Velocity Saturation (cont'd)

- Assume  $V_{DSAT} = \xi_c L/2$  when  $(V_{GS} - V_T) > \xi_c L/2$

$V_{DSAT}$  (V)

$\xi_c L/2$

$V_{GS} - V_T$  (V)

Actual  $V_{DSAT}$

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### One Last Simplification

- If device always operates in velocity sat.:

$$I_D = k' \cdot \frac{W}{L} \cdot \left( V_{GS} - V_T - \frac{V_{D,VSAT}}{2} \right) V_{D,VSAT}$$

- " $V_T^*$ " model:

$$V_T^* = V_T + \frac{V_{D,VSAT}}{2}$$

$$I_D = k' \cdot \frac{W}{L} \cdot (V_{GS} - V_T^*) V_{D,VSAT}$$

- Good for first cut, simple analysis

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### Simplified Model

- Define  $V_{GT} = V_{GS} - V_T$ ,  $V_{D,VSAT} = \xi_c \cdot L/2$

$I_D$  (A)

$V_{DS}$  (V)

Linear

Velocity Saturation

Saturation

$V_{DS} = V_{D,VSAT}$

$V_{GT} = V_{D,VSAT}$

$V_{DS} = V_{GT}$

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### Next Lecture

- Using the MOS model: VTCs

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