Announcement

- Discussion Notes for this week contain another review of mixers, highly recommended!
- Review OH notes cover mixers, system design and transformers, make sure to go through (on bspace)
- Tomorrow (Rev OH) will go through transformers/ Noise in Mixer/ Examples (as time permits)
**Balanced Mixer Operation**

- The AC operation nearly identical to a single balanced Gilbert cell.
- Even a single ended output, though, effectively multiplies the signal by ±1, thus rejecting the RF signal.

**Output Signal**

- Vs driving between +RF and -RF

\[
I_{c5} = \frac{I_{EE}}{2} + i_s
\]

\[
I_{c6} = \frac{I_{EE}}{2} - i_s
\]

\[
i_s = \frac{1}{2}v_s \left( \frac{g_{m5,6}}{1 + g_{m5,6}R_E} \right)
\]

\[
s(t) = \frac{4}{\pi} \cos(\omega_0 t) - \frac{1}{3\pi} \cos(3\omega_0 t) + ...
\]

\[
I_0 = \frac{I_{EE}}{2} + i_s s(t)
\]

\[
IF_{out} = \frac{1}{2} \left( \frac{g_{m5,6}}{1 + g_{m5,6}R_E} \right) v_s \frac{2}{\pi} \cos(\omega_s \pm \omega_0 t)
\]
LO Rejection

- As before, the RF stage is a transconductance stage. Degeneration can be used to linearize this stage.
- Because the bias current at the output is constant $I_{EE}/2$ regardless of the LO voltage, the LO signal is rejected. This relies on good matching between transistors.
- The differential operation also rejects even order distortion. Viewed as two parallel Gilbert cells, this mixer is also more linear as it processes only half of the signal.
- The noise of this mixer, though, is higher since the noise in each transistor is independent (come back to this point later, also see HW)

Mixer Analysis: Time Domain

- A generic mixer operates with a periodic transfer function $h(t + T) = h(t)$, where $T = 1/\omega_0$, or $T$ is the LO period. We can thus expand $h(t)$ into a Fourier series

$$y(t) = h(t)x(t) = \sum_{-\infty}^{\infty} c_n e^{j\omega_0 nt}x(t)$$

- For a sinusoidal input, $x(t) = A(t) \cos \omega_1 t$, we have

$$y(t) = \sum_{-\infty}^{\infty} \frac{c_n}{2} A(t) \left( e^{j(\omega_1 + \omega_m)t} + e^{j(-\omega_1 + \omega_m)t} \right)$$

- Since $h(t)$ is a real function, the coefficients $c_{-k} = c_k$ are even. That means that we can pair positive and negative frequency components.
Time Domain Analysis

- Take $c_1$ and $c_{-1}$ as an example

$$
= c_1 \frac{e^{j(\omega_1 + \omega_0)t}}{2} A(t) + c_1 \frac{e^{j(\omega_1 - \omega_0)t}}{2} A(t) + \cdots
$$

$$
= c_1 A(t) \cos(\omega_1 + \omega_0)t + c_1 A(t) \cos(\omega_1 - \omega_0)t + \cdots
$$

- Summing together all the components, we have

$$
y(t) = \sum_{-\infty}^{\infty} c_n \cos(\omega_1 + n\omega_0)t
$$

- Unlike a perfect multiplier, we get an infinite number of frequency translations up and down by harmonics of $\omega_0$.

Frequency Domain Analysis

- Since multiplication in time, $y(t) = h(t)x(t)$, is convolution in the frequency domain, we have

$$
Y(f) = H(f) \ast X(f)
$$

- The transfer function $H(f) = \sum_{-\infty}^{\infty} c_n \delta(f - nf_0)$ has a discrete spectrum. So the output is given by

$$
Y(f) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} c_n \delta(\sigma - nf_0) X(f - \sigma) d\sigma
$$

$$
= \sum_{-\infty}^{\infty} c_n \int_{-\infty}^{\infty} \delta(\sigma - nf_0) X(f - \sigma) d\sigma
$$
Frequency Domain (Folding)

- By the frequency sifting property of the $\delta(f - \sigma)$ function, we have
  \[ Y(f) = \sum_{-\infty}^{\infty} c_n X(f - nf_0) \]
- Thus, the input spectrum is shifted by all harmonics of the LO up and down in frequency.

Harmonic Mixer

- We can use a harmonic of the LO to build a mixer.
- Example, let $LO = 500\text{MHz}$, $RF = 900\text{MHz}$, and $IF = 100\text{MHz}$.
- Note that $IF = 2LO - RF = 1000 - 900 = 100$
Harmonic Mixer Conversion Gain

- The $n$th harmonic conversion transconductance is given by
  \[ g_{\text{conv},n} = \frac{|\text{IF current out}|}{|\text{input signal voltage}|} = \frac{g_m}{2} \]

- For a BJT, we have
  \[ g_{\text{conv},n} = g_m Q \frac{I_n(b)}{I_0(b)} \]

- The advantage of a harmonic mixer is the use of a lower frequency LO and the separation between LO and RF.
- The disadvantage is the lower conversion gain and higher noise.

Distortion In Mixers

- Using the same formulation, we can now insert a signal with two tones
  \[ v_{in} = V_{s1} \cos \omega_{s1} t + V_{s2} \cos \omega_{s2} \]
  \[ I_C = I_S e^{V_A/V_t} \times e^{b \cos \omega_0 t} \times e^{V_{s1} \cos \omega_{s1} t + V_{s2} \cos \omega_{s2} t} \]

- The final term can be expanded into a Taylor series
  \[ I_C = I_S e^{V_A/V_t} \times e^{b \cos \omega_0 t} \times \left(1 + V_{s1} \cos \omega_{s1} t + V_{s2} \cos \omega_{s2} 2t + \cdots\right) \]

- The square and cubic terms produce $IM$ products as before, but now these products are frequency translated to the IF frequency.
**Noise In Mixers**

- By definition we have $F = \frac{SNR}{SNR_0}$. If we apply this to a receiving mixer, the input signal is at the “RF” and the output signal is at “IF”. There is some ambiguity to this definition because we have to specify if the RF signal is a single or double sideband modulated waveform.

- For a double sideband modulated waveform, there is signal energy in both sidebands and so for a perfect multiplying mixer, the $F = 1$ since the IF signal is twice as large since energy from both sidebands fall onto the IF. For a single-sideband modulated waveform, though, the noise from the image band adds doubling the IF noise relative to RF. Thus the $F = 2$.

- If an image reject filter is used, the noise in the image band can be suppressed and thus $F = 1$ for a cascade of a sharp image reject filter followed by a multiplier.

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**Mixer Noise Folding**

- Since the mixer will downconvert any energy at a distance of $IF$ from the LO and it’s harmonics, all the noise from these image bands will be downconverted to the same IF.

- For a Gilbert cell type mixer, the current of the $G_m$ stage produces white noise which is downconverted. Summing over all the harmonics, we have

$$\sum_{k=-\infty}^{\infty} |c_k|^2 = \frac{1}{T} \int_{0}^{T} s(t)^2 dt$$

- where the last equality follows from Parseval's Theorem.

- For a square waveform $s(t)$, the harmonic powers fall like $1/k^2$. Thus the third harmonic contributes about 10% to the switching pair noise.
Mixer Noise from Switching Pair

- In addition to the noise folding discussed above, the switching pair itself will contribute noise at IF. If Q1/Q2 are both on, they act as a differential amplifier and introduce noise.

- When only Q1 or Q2 is on, though, the noise is rejected due to the degeneration provided by the transconductance device.

- Thus we generally use a large LO signal to minimize the time when both devices are conducting. It’s therefore not surprising that the noise figure of the mixer improves with increasing LO amplitude.

Voltage Switching Mixers

- Instead of switching currents, we can also switch the voltage.

- In the above circuit, during the $+LO$ cycle, switch $S1$ activates and feeds the RF to the output directly. In the $-LO$ cycle, switch $S2$ activates and feeds an inverted RF signal to the output.

- This circuit requires good switches that turn on hard (low on-resistance) and turn off well (good isolation).
MOS Voltage Switching Mixer

- A practical implementation uses MOS devices as switches. The devices are large to minimize their on-resistance with a limit determined by isolation (feed-through capacitance).

- We see that the RF signal is effectively multiplied by ±1 with a rate determined by the LO signal.

- A differential RF signal is created using a balun or fed directly from a balanced LNA.

Non-Ideal Switches

- When the device is "on", it's in the triode region. Due to the low on-resistance, the coupling through the substrate and LO path is minimal.

- When the device is "off", the RF and LO leak into the IF through the overlap and substrate capacitances.
MOS Switching (Passive) Mixer Summary

- MOS passive mixer is very linear. The device is either “on” or “off” and does not impact the linearity too much. Since there is no transconductance stage, the linearity is very good.
- The downside is that the MOS mixer is passive, or lossy. There is no power gain in the device.
- Need large LO drive to turn devices on/off
- Need to create a differential RF and LO signal. This can be done using baluns or by using a differential LNA and LO buffer.

MOS Ring Mixer

- The RF/LO/IF are all differential signals. During the positive LO cycle, the RF is coupled to the IF port with positive phase, whereas during the negative phase the RF is inverted at the IF.
- The MOS resistance forms a voltage divider with the source and load and attenuates the signal as before.
Passive Mixer LO Power

- Since gates of the MOS switches present a large capacitive load, a buffer is needed to drive them.
- The LO buffer can be realized using larger inverters (approach "square wave") or as a tuned buffer. A tuned lowers the power by roughly $Q$ but has a sinusoidal waveform.

LO Power (Inverter Chain)

- For an inverter chain driving the LO port, the power dissipation of the last stage is given by

$$P_{inv} = CV_{LO}^2 f_{LO}$$

- $C$ is the total load presented to the LO (two MOS devices for the double balanced mixer, one MOS device for single balanced).
- $V_{LO}$ is the LO amplitude to fully turn the devices on and off. The devices should be biased near threshold. $f_{LO}$ is the LO frequency.
**Tuned LO Power**

- For the tuned load case, the power is given by
  \[ P_{tuned} = \frac{V_{LO}^2}{2R_t} \]

- \( R_t \) is the effective shunt resistance of the tank. Since the tank
  \( Q = \omega_{LO} R_t C \), we have
  \[ P_{tuned} = \frac{V_{LO}^2}{2Q} \omega_{LO} C = \frac{\pi C V_{LO}^2 f_{LO}}{Q} \]

- A high \( Q \) tank helps to reduce the power consumption of the LO buffer.

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**H-Bridge Mixer**

- If PMOS devices are available, two CMOS inverters form an H-Bridge, applying the RF input signal to the IF directly during the \( LO \) cycle and inverting the RF input at the IF output in the \( \overline{LO} \) cycle.

- PMOS and NMOS devices are sized appropriately to maximize on-conductance and to minimize off capacitance.
**Time-Varying Conductance**

- The RF voltage is applied to a time varying conductance. Note that if the conductance of the LO switches is given by \( g(t) \), then the conductance of the LO switches is given by \( g(t - T_{LO}/2) \).

- The Thevenin equivalent source voltage is given by the open circuit voltage

\[
v_T = v_{RF} \left( \frac{g(t)}{g(t) + g(t - T_{LO}/2)} - \frac{g(t - T_{LO}/2)}{g(t) + g(t - T_{LO}/2)} \right)
\]

\[
v_T = v_{RF} \left( \frac{g(t) - g(t - T_{LO}/2)}{g(t) + g(t - T_{LO}/2)} \right) = m(t) v_{RF}
\]

**Time-Varying Gain \( m(t) \)**

- For the MOS device and a given LO waveform, the function \( m(t) \) can be calculated and the Fourier expansion can be used to derive the gain.

- In practice there is a load capacitance \( C_{IF} \) at the IF port to filter the downconverted signal. This \( C_L \) complicates the analysis but interested students are encouraged to read the paper by Shahani, Shaefier and Lee (JSSC vol. 32, Dec 1997, p. 2061-1071)
Passive Current Mixer

- The input stage is a $G_m$ stage similar to a Gilbert cell mixer. The Gilbert Quad, though, has no DC current and switches on/off similar to a passive mixer.
- The output signal drives the virtual ground of a differential op-amp. The current signal is converted into a voltage output by the op-amp.

Advantages of Passive Current Mixer

- No DC current in quad implies that there is no flicker noise generated by the switching quad. This is the key advantage.
- The linearity is very good since the output signal is a current. The voltage swing does not limit the linearity of the mixer. This is to be contrasted to a Gilbert cell mixer where the voltage swing is limited due to the headroom of the switching mixer and the transconductance stage.
- The op-amp output stage can be converted into an IF filter (discussed later)
Disadvantages

- Need large LO drive compared to the active Gilbert cell mixer.
- Need an op-amp. This requires extra power consumption and introduces additional noise.
- Need a common mode feedback circuit at the input of the op-amp.

Ring or Quad?

- Note that the Gilbert quad is really a folded ring. Thus the passive and active mixers are very similar. The main difference is how the quad devices are biased. In the Gilbert cell they are biased nominally in saturation and have DC current. In the passive mixers, they are biased near the threshold.
Op-Amp Noise

- The op-amp input referred noise is amplified to IF. The resistance seen at the op-amp input terminals is actually a switched capacitor resistor.
- The parasitic capacitance at the output of the transconductance stage is charged and discharged at the rate of the LO.

Switched Capacitor Noise

- Note that the parasitic capacitances are charged at the rate of the LO to the input voltage $V_x$, and then to the $-V_x$, every cycle.
- The total charge transferred during a period is given by
  \[ Q_{tot} = C_p V_x - (-C_p V_x) = 2C_p V_x \]
- The net current is given by
  \[ I_x = \frac{Q_{tot}}{T_{LO}} = 2C_p V_x f_{LO} \]
Switched Capacitor Resistor

- Since there are two differential pairs connected to the op-amp terminals in parallel, the total charge is twice. So the effective resistance seen at this node is given by
  \[ R_p = \frac{V_x}{2I_x} \]

- The effective resistance is therefore given by
  \[ R_p = \frac{1}{4f_{LO}C_p} \]

- This is a switched capacitor "resistors".

Op-Amp Noise

- The noise is thus transferred to the output with transfer function given by
  \[ \overline{v_o^2} = \left( 1 + \frac{2R_f}{R_p} \right) \overline{v_{amp}^2} \]

- To minimize this noise, we have to minimize the parasitic capacitance \( C_p \) and the op-amp noise.
Output Filter Stage

- Since a down-conversion mixer will naturally drive a filter, we see that the output current can be used directly to drive a current mode filter.
- For instance, the op-amp can be absorbed into the first stage of a multi-stage op-amp RC IF filter. The feedback resistor $R_f$ is shunted with a capacitor $C_f$ to produce a pole.

APPENDIX I:
PERIODIC STEADY STATE SIMULATIONS
Also refer to Discussion notes of 11/08/2010
Periodic Steady State (PSS) Simulations

- Transient simulation is slow and costly because we have to do a tight tolerance simulation of several IF cycles with a weak RF.
- The SpectreRF PSS analysis is a tool for finding the periodic steady-state solution to a circuit. In essence, it tries to find the initial condition or state for the circuit (capacitor voltages, inductor currents) such that the circuit is in periodic steady state.
- It can usually find the periodic solution within 4-5 iterations.
- In the mixer, if we ignore the RF signal, then the periodic operating point is determined by the LO signal alone.

PSS Iteration

- Since typical PSS run converges in 4-5 cycles of the LO, or a simulation time of about $5T_{LO}$, the overall simulation converges several orders of magnitude faster than transient at the IF frequency.
- PSS requires that the circuit is not chaotic (periodic input leads to a periodic output).
- High Q circuits do not pose a problem to PSS simulation since we are finding the steady-state solution. The high Q natural response takes roughly $Q$ cycles to die down, thus saving much simulation time.
PSS Options

- We can perform PSS analysis on driven or autonomous circuits. An autonomous circuit has no periodic inputs but produces a periodic output (e.g. an oscillator).
- For PSS analysis we need to specify a list of “large” signals in the circuit. In a mixer, the only large tone is the LO, so there is only one signal to list.
- We also specify the “beat frequency” or the frequency of the resulting periodic operating point. For instance if we drive a circuit with two large tones at $f_1$ and $f_2$, the beat frequency is $|f_1 - f_2|$. Spectre can auto-calculate this frequency.
- An additional time for stabilization $t_{stab}$ can be specified to help with the convergence. For a mixer this is not needed.

Periodic AC (PAC) Simulations

- Once a PSS analysis is performed at the “beat frequency”, the circuit can be linearized about this time varying operating point.
- Note that for a given AC input, there are as many transfer functions as there are harmonics in the LO.
- We specify the frequency range of the AC input signal as either an absolute or relative range. A relative range is a frequency offset with respect to the beat frequency.
- We also specify the maximum sidebands to keep for the simulation. Note that this does not affect the accuracy of the simulation but simply the amount of saved data.
PAC

- The input frequency $f_i$ is translated to frequencies $f_i + k f_o$, where $f_o$ is the beat (LO) frequency. The $k = 0$ sideband corresponds to the DC component of the LO signal (e.g. time invariant behavior). The non-zero components, though, correspond to mixing. E.g. $k = -1$ correspond to frequency down-conversion. $k = +1$ is the normal up-conversion. $k = -2$ is the 2nd harmonic mixing $f_i - 2f_o$.

PNoise

- PNoise is a noise analysis that takes the frequency translation effects into account. The simulation parameters are similar to PAC with the exception that we must identify the input and output ports (for noise figure) and the reference side-band, or the desired output frequency. For a mixer, this is $k = -1$. 