Lecture 8: Power Gain/ Matching Networks

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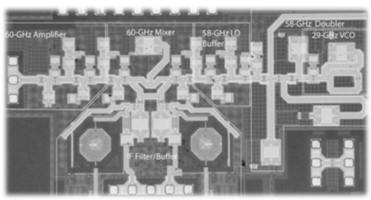

EE142 – Fall 2010

Sept. 21st, 2010

Announcements

- Postlab1 due date postponed to 28th September
- HW3 is due Thursday, 23rd September (Collected at beginning of class, 330pm+10min)
- *Review OH* schedules will be posted (will cover feedback and ?)
- Wednesday 10-11am for “review” OH?

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Outline

- Last Lecture: Two Ports
  - Transistor Y-Parameters
  - Input/output admittance, voltage gain,…
  - Feedback and extracting feedback parameters
  - Stability
  - K-Factor
- This Lecture: Power Gain and Matching (Based on Prof. Niknejad's notes and book chapter)
  - Power gain vs voltage gain
  - Available power from a source
  - Various forms of power gain
  - Matching networks

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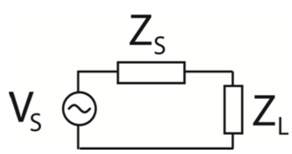
Available Power from a Source



- Maximum power we can extract from source (given fixed source impedance)
- But we know conjugate matching is the best power extraction scheme:
- $$\underbrace{P}_{Matched\ Load} = \frac{1}{2}Re(VI^*) = \frac{1}{2}Re\left(\frac{V_S}{2}\left(\frac{V_S}{2Re(Z_S)}\right)^*\right)$$
  $$= \frac{|V_S|^2}{8Re(Z_S)} = P_{av,s}$$

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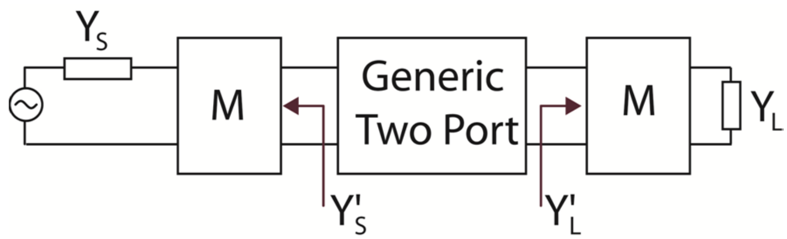
How To Obtain Maximum Power Gain



- Now that we know *power* is important how can we obtain the maximum power gain
  - Power Gain Metrics
  - Similarity to maximum power transfer theorem

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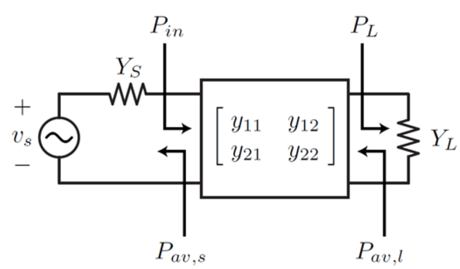
Power Gain



$$P_{in} = \frac{|V_1|^2}{2}\Re(Y_{in})$$

$$P_L = \frac{|V_2|^2}{2}\Re(Y_L)$$

$$G_p = \left|\frac{V_2}{V_1}\right|^2 \frac{\Re(Y_L)}{\Re(Y_{in})}$$

$$G_p = \frac{|Y_{21}|^2}{|Y_L + Y_{22}|^2}\frac{\Re(Y_L)}{\Re(Y_{in})}$$

- We can define power gain in many different ways. The *power gain* or *operating power gain* Gp is defined:

$$G_p = \frac{P_L}{P_{in}} = f(Y_L, Y_{ij}) \neq f(Y_S)$$

- We note that this power gain is a function of the load admittance Y_L and the two-port parameters Y_{ij}.

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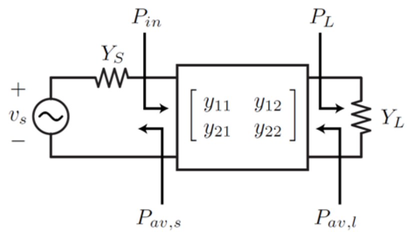
Available and *Transducer* Power Gains



Available Power Gain: $$G_a = \frac{P_{av,L}}{P_{av,S}} = f(Y_S, Y_{ij}) \neq f(Y_L)$$

Transducer Power Gain: $$G_T = \frac{P_L}{P_{av,S}} = f(Y_L, Y_S, Y_{ij})$$

- Four variables: P_L, P_{in}, $P_{av,s}$, $P_{av,L}$
- What is the relationship between these three power gains? Can they be equal? Under what conditions?

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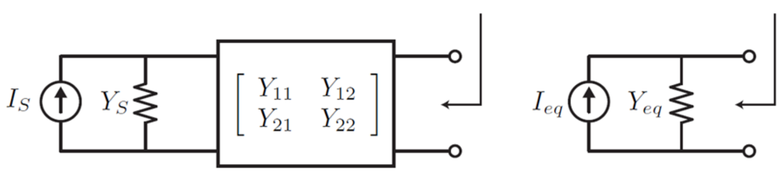
Derivation of Available Power Gain



- The source and the two-port together form a “source”. What is the available power from this source?
- Norton equivalent circuit:

1- Short the outputs → Derive I_{eq}

- $I_{eq} = -I_2 = -Y_{21} \frac{I_S}{Y_{11}+Y_S}$

2- “Remove” the source, look at the output admittance (this was derived in the last lecture):

$$Y_{eq} = Y_{22} - \frac{Y_{21}Y_{12}}{Y_{11}+Y_S}$$

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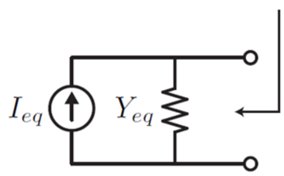
Deriving Available Power Gain:



$$P_{av,L} = \frac{|I_{eq}|^2}{8\Re(Y_{eq})}$$

$$P_{av,S} = \frac{|I_S|^2}{8\Re(Y_S)}$$

$$G_a = \frac{|Y_{21}|^2}{|Y_{11} + Y_S|^2} \frac{\Re(Y_S)}{\Re(Y_{eq})}$$

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Transducer Power Gain (G_T)

- How much better are we doing than just a matched load?
- As expected is a function of both load and source impedances
- How much power we actually deliver to our load/ Maximum power our source could provide

$$G_T = \frac{P_L}{P_{av,S}} = \frac{\frac{1}{2}\Re(Y_L)|V_2|^2}{\frac{|I_S|^2}{8\Re(Y_S)}} = 4\Re(Y_L)\Re(Y_S)\left|\frac{V_2}{I_S}\right|^2$$

Deriving V_2/I_s :

$$\left|\frac{V_2}{V_1}\right| = \left|\frac{Y_{21}}{Y_L + Y_{22}}\right|$$

$$I_S = V(Y_S + Y_{in})$$

$$\left|\frac{V_2}{I_S}\right| = \left|\frac{Y_{21}}{Y_L + Y_{22}}\right| \frac{1}{|Y_S + Y_{in}|}$$

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Transducer Power Gain (2)

- But we know:

$$|Y_S + Y_{in}| = \left| Y_S + Y_{11} - \frac{Y_{12}Y_{21}}{Y_L + Y_{22}} \right|$$

- and therefore:

$$\left| \frac{V_2}{I_S} \right|^2 = \frac{|Y_{21}|^2}{|(Y_S + Y_{11})(Y_L + Y_{22}) - Y_{12}Y_{21}|^2}$$

- The Transducer Power Gain:

$$G_T = \frac{4\Re(Y_L)\Re(Y_S)|Y_{21}|^2}{|(Y_S + Y_{11})(Y_L + Y_{22}) - Y_{12}Y_{21}|^2}$$

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Comparison of Power Gains

- It's interesting to note that *all* of the gain expression we have derived are in the exact same form for the impedance, hybrid, and inverse hybrid matrices.
- In general, $P_L \leq P_{av,L}$, with equality for a matched load. Thus we can say that
  - $G_T \leq G_a$
- The maximum transducer gain as a function of the load impedance thus occurs when the load is conjugately matched to the two-port output impedance

$$G_{T,max,L} = \frac{P_L(Y_L = Y_{out}^*)}{P_{av,S}} = G_a$$

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Comparison of Power Gains (2)

- Likewise, since $P_{in} \leq P_{av,S}$, again with equality when the the two-port is conjugately matched to the source, we have
  - $G_T \leq G_p$
- The transducer gain is maximized with respect to the source when

$$G_{T,max,S} = G_T(Y_{in} = Y_S^*) = G_p$$

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Maximum Power Gain

- Intuitively observed that we need input and output match for best power transfer (Bi-Conjugate Match)

$$Y_{in} = Y_{11} - \frac{Y_{12}Y_{21}}{Y_L + Y_{22}} = Y_S^*$$

$$Y_{out} = Y_{22} - \frac{Y_{12}Y_{21}}{Y_S + Y_{11}} = Y_L^*$$

- The rigorous proof starts with:

$$\frac{\partial G_T}{\partial G_S} = \frac{\partial G_T}{\partial B_S} = \frac{\partial G_T}{\partial G_L} = \frac{\partial G_T}{\partial B_L} = 0$$

- To simplify we can look at Gp and Ga separately

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Maximum Power Gain Derivation

- Refer to Prof. Niknejad's book chapter for derivation

$$Y_{jk} = m_{jk} + jn_{jk}$$

$$Y_L = G_L + jX_L$$

$$Y_{12}Y_{21} = P + jQ = Le^{j\phi}$$

⇒

$$B_{L,opt} = \frac{Q}{2m_{11}} - n_{22}$$

$$G_{L,opt} = \frac{1}{2m_{11}}\sqrt{(2m_{11}m_{22} - P)^2 - L^2}$$

Requires that: $K = \dfrac{2m_{11}m_{22} - P}{L} > 1$ We've seen this condition before

$$G_{p,max} = \frac{|Y_{21}|^2}{2m_{11}m_{22} - P + \sqrt{(2m_{11}m_{22} - P)^2 - L^2}}$$

$$G_{p,max} = G_{T,max} = G_{a,max} = G_{max} = \frac{Y_{21}}{Y_{12}}(K - \sqrt{K^2 - 1})$$

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Maximum Power Gain of a Unilateral Two-Port

$$G_T = \frac{4\Re(Y_L)\Re(Y_S)|Y_{21}|^2}{|(Y_S + Y_{11})(Y_L + Y_{22}) - Y_{12}Y_{21}|^2}$$

$$Y_S = Y_{11}^*$$

$$Y_L = Y_{22}^*$$

$$G_{T,max} = \frac{|Y_{21}|^2}{4m_{11}m_{22}}$$

- Sometimes called the Maximum Available Gain or MAG for a two-port

For a non-unilateral two-port with K<1 MSG (Maximum Stable Gain) is defined as Gmax with K=1:

$$G = \left|\frac{Y_{21}}{Y_{12}}\right|$$

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Example: Power Gain of a Unilateral MOS

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Why Match?

- Maximum *Power* transfer
- Reduce reflections, re-radiation, voltage "nulls"…
- Non-impedance controlled environment (50 Ohm setting)
- Noise and Efficiency require optimal loads

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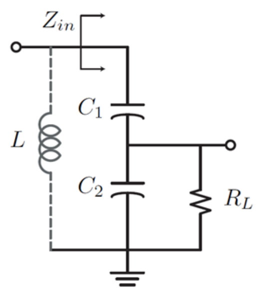
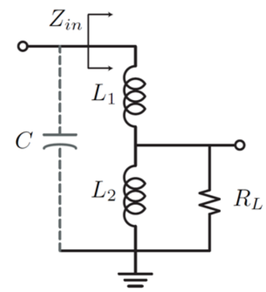
Capacitive/Inductive Dividers





$$v_o = v_{C_2} = jX_2 \times i = v_i \frac{X_2}{X_1 + X_2} = v_i \frac{1}{1 + \frac{C_2}{C_1}} = kv_i$$

- Loss-less network→ Current also scales:

$$R_{in} \approx \left(1 + \frac{C_2}{C_1}\right)^2 R_L$$

- We can also derive this using Series-Parallel transformation

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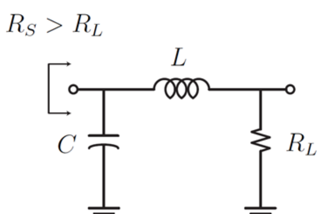
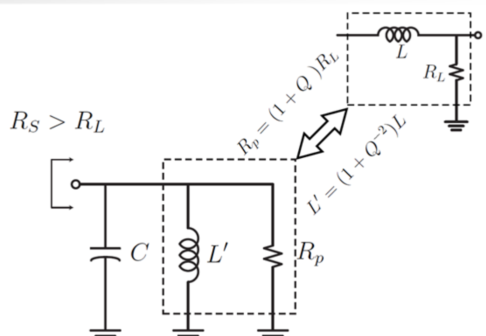
L-Match





- We can step-up or step-down the impedance by using series-shunt or shunt-series networks (L-Match)

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Matching Network Design

1. Calculate the boosting factor $m = \frac{R_{hi}}{R_{lo}}$
2. Compute the required circuit Q by
   $(1 + Q^2) = m$, or $Q = \sqrt{m-1}$.
3. Pick the required reactance from the Q. If you're boosting the resistance, e.g. $R_S > R_L$, then $X_S = Q \cdot R_L$. If you're dropping the resistance, $X_p = R_L / Q$
4. Compute the effective resonating reactance. If $R_S > R_L$, calculate $X'_s = X_s(1 + Q^{-2})$ and set the shunt reactance in order to resonate, $X_p = -X'_s$. If $R_S < R_L$, then calculate $X'_p = X_p/(1+Q^{-2})$ and set the series reactance in order to resonate, $X_s = -X'_p$.
5. For a given frequency of operation, pick the value of L and C to satisfy these equations.

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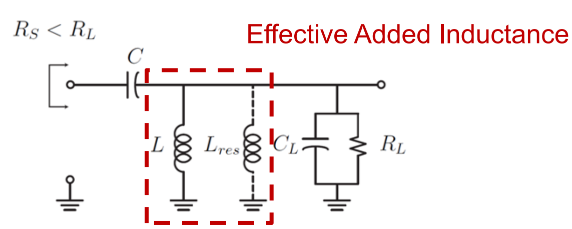
Complex Source/ Load

- First "absorb" the extra reactance/ susceptance
- We can then move forward according to previous guidelines



- There might be multiple ways of achieving matching, each will have different properties in terms of BW (Q), DC connection for biasing, High-pass vs Low-Pass,…

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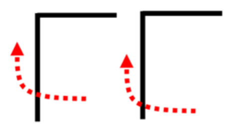
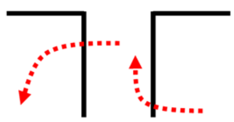
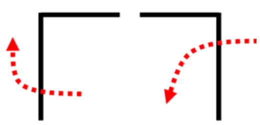
Multi-Stage Matching Networks



1. Cascaded L-Match
   - Wide bandwidth
   - Only in one direction



- T-Match
  - First transform high then low
  - BW is lower than single L-Match



- Pi-Match
  - First low then high
  - BW is lower than single L-match

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