Lecture 3:
Transistors, Amplifiers and
Bandwidth Limitations

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Introduction
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At a glance

- Previous lecture: modulation
- Today: transistors, amplifiers

Amplifiers in Transceivers

- Used in many functions ...
- Key requirement: bandwidth
  - What limits the bandwidth?

CMOS Cross Section

- Modern short channel CMOS process has very short channel lengths ($L < 100 \text{ nm}$). To ensure gate control of channel, as opposed to drain control (DIBL), we employ thin junctions and thin oxide ($t_{ox} < 5 \text{ nm}$).
- Due to lithographic limitations, there is an overlap between the gate and the source/drain junctions. This leads to overlap capacitance. In a modern FET this is a substantial fraction of the gate capacitance.

Small Signal Model

- The junctions of a FET form reverse-biased p-n junctions with the substrate (well), or the body node. This is another form of parasitic capacitance in the structure, $C_{gs}$ and $C_{gd}$.
- At low frequency, $R_{ds} \approx \infty$. There is external gate resistance $R_g$ due to the polysilicon gate and $R_d$ due to junction resistance.
- In the forward active (saturation) region, the input capacitance is given by $C_{in} = \frac{1}{2} C_{ox} \cdot W \cdot L$.
- $r_o$ is due to channel length modulation and other short channel effects (such as DIBL).

Simplified Small Signal Model

- You’re probably more familiar with the simplified FET model without gate and source resistance. These resistances are small but are important when we analyze the power gain of the device.
- If we ground the bulk node, such as a common source amplifier, we can eliminate a lot of clutter since the $g_{ds}$ generator is shorted.
MOS Device Characteristics

- For a long channel FET, before “pinch-off”, the device drain current responds nearly linearly with \( V_{GS} \), hence the term “linear region”.

\[
I_{DS} = \frac{W}{L} \rho C_{ox} \left( V_{GS} - V_T \right) \left( V_{DS} - \frac{V_T}{2} \right)
\]

- Beyond pinch-off, when \( V_{GS} = V_{GS} - V_T \), the current saturates and remains essentially constant. This is the saturation region.

\[
I_{DS} = \frac{1}{2} \frac{W}{L} \rho C_{ox} \left( V_{GS} - V_T \right)^2 \left( 1 + \frac{V_T}{2V_{DS}} \right)
\]

Unity Gain Frequency

- The short circuit current gain of a device is given by

\[
G_i = \frac{i_e}{i_0} = \frac{g_m}{\beta (C_{es} + C_{gd})}
\]

- The frequency of unity gain \( \omega_T \) is given by solving \( |G_i| = 1 \)

\[
\omega_T = \frac{g_m}{C_{es} + C_{gd}}
\]

- This frequency plays an important role in the frequency response of high speed amplifiers. Often there is a gain-bandwidth tradeoff related to \( \omega_T \)

\[ G \times BW = \omega_T \]

MOS Unity Gain Frequency

- For a long-channel MOSFET we have the following relationship

\[
\omega_T = \frac{g_m}{C_{es} + C_{gd}} = \frac{\rho C_{ox} \left( V_{GS} - V_T \right)}{2W/L \mu C_{ox}}
\]

- Canceling common factors we have

\[
\omega_T = \frac{3}{2} \frac{\rho C_{ox} \left( V_{GS} - V_T \right)}{L}
\]

- We see that \( \omega_T \) is bias dependent. The strong \( L^2 \) length dependence only holds for long-channel devices. Short channel devices, in the limit of velocity saturated operation, reduce to \( 1/L \) dependence.

- Note that \( \rho = f \) (\( V_{GS} - V_T \)) due to reduced mobility at the surface of the transistor.

Modern Bipolar Device

- Most transistor “action” occurs in the small npn sandwich under the emitter. The base width should be made as small as possible in order to minimize recombination. The emitter doping should be much larger than the base doping to maximize electron injection into the base.

- A SiGe HBT transistor behaves very similarly to a normal BJT, but has lower base resistance, since the doping in the base can be increased without compromising performance of the structure.

Bipolar “Gain”

- Due to Boltzmann statistics, the collector current is described very accurately with an exponential relationship

\[
I_C = I_{C0} \frac{V_{CE}}{V_T}
\]

- The device transconductance is therefore proportional to current

\[
g_m = \frac{dI_C}{dV_{CE}} = I_C \frac{2}{kT} \frac{V_{CE}}{V_T} = \frac{qI_C}{kT}
\]

- where \( kT/q = 26 \text{ mV} \) at room temperature. Compare this to the equation for the FET. Since we usually have \( kT/q < (V_{GS} - V_T) \), the bipolar has a much larger transconductance for the same current. This is the biggest advantage of a bipolar over a FET.
Small Signal Model

- The core model is similar to a FET small-signal model. The resistor $r_e$, though, dominates the input impedance at low frequency. At high frequency, $C_e$ dominates.
- $C_c$ is due to the collector-base reverse biased diode capacitance. $C_{tr}$ is the collector to substrate parasitic capacitance. In some processes, this is reduced with an oxide layer.
- $C_v$ has two components, due to the junction capacitance (forward-biased) and a diffusion capacitance:
  $$C_v = C_{v(f)} + C_{v(dif)}$$

Generic Small Signal Model (High Freq.)

- The generic figure above represents both a FET and a bipolar at high frequency.
- Notice that this model holds when $r_e \gg 1/X_{ce}$.
- Since
  $$\frac{r_e}{X_{ce}} = \omega r_e C_e = \frac{\beta h_{fe} C_e}{\omega} = \beta \omega$$
- Say $\beta = 100$ and the operating frequency is $\omega/\omega_T = 1/10$. Then we have $r_e/X_{ce} = 100/10 = 10$.

Bipolar Unity Gain Frequency

- Similar to a FET we have the following relationship:
  $$\omega_T = \frac{g_m}{C_b + C_e^l}$$
- Expanding the denominator term:
  $$\omega_T = \frac{g_m}{C_b + C_e + C_e^l} = \frac{g_m}{2C_e + \frac{g_m}{2C_e^l} + g_m\tau_T + C_e^l}$$
- The collector junction capacitance is a function of $V_{ce}$ or the reverse bias. To maximize $\omega_T$, we should maximize the collector voltage. Re-writing the above equation:
  $$\omega_T = \frac{1}{\tau_T + \frac{2C_e + C_e^l}{g_m}}$$

Bias Dependence

- We can clearly see that if we continue to increase $I_C$, then $g_m = I_C$ increases and the limiting value of $\omega_T$ is given by the forward transit time $\tau_T$:
  $$\omega_T = \frac{1}{\tau_T}$$
- In practice, though, we find that there is an optimum collector current. Beyond this current the $\omega_T$ drops. This optimum point occurs due to the Kirk Effect. It’s related to the “base widening” due to high level injection.

Single Stage Amplifiers

- The CE/CS amplifier has a small bandwidth due to the Miller feedback. We must keep the source resistance low.
- The CC/CD (or follower) is very wideband but only provides current gain.
- The CB/CG amplifier is also wideband (no Miller), but only offers voltage gain. Has small input impedance (sometimes good).
- The CE/CS offers the best power gain and noise figure, but bandwidth limitations are an issue.

Basic Amplifiers and their Small Signal Models
A Broadband Amplifier: Current Amplifier

- A 1 x N current mirror has broadband frequency response which can be illustrated with the equivalent circuit. The diode-connected device can be replaced with a conductance of value $g_{m}$ in shunt with the amplifier input capacitance $C_{in}$.
- If the current amplifier drives a low impedance load, the transfer function is given by

$$G_{t} = \frac{g_{m} \cdot \frac{g_{m2}}{g_{m1}}}{1 + (N+1)g_{m2}}$$

Current Amplifier Analysis

- Note that the transconductance of output device is $N$ times larger since it can be thought of $N$ devices in parallel. The complete transfer function is therefore

$$G_{t} = \frac{N}{1 + \frac{1}{g_{m2}/g_{m1}}(N+1)}$$

and the gain-bandwidth product is given by

$$G_{t} \cdot \omega_{p} = \frac{N}{1 + \frac{1}{g_{m2}/g_{m1}}} \cdot \omega_{p}$$

- It’s important to note that the above analysis holds only if we assume the load impedance is extremely low, ideally a short. If we connect a physical resistor to the output, the Miller effect will produce a significant feedback current which invalidates our assumptions.
- Note amplifier is large signal linear.

CS/CE Bandwidth Limitation

- Due to Miller multiplication, the input cap is usually the dominant pole

$$\omega_{p} = \frac{1}{R_{L}C_{in} + |A_{i}|C_{i}}$$

$$\omega_{p} = \frac{1}{R_{L}C_{in} + 1 + |A_{i}|} = R_{L}C_{in}|A_{i}|$$

CS/CE Gain-Bandwidth Product

- Assuming the voltage gain is given by the low-frequency value of $g_{m}R_{L}$, we have

$$\omega_{0}^{-1} = R_{L}C_{in}g_{m}R_{L} = (g_{m}R_{L})\frac{C_{in}}{g_{m}}$$

$$\omega_{0}^{-1} = \frac{|A_{i}|^{2}R_{L}}{R_{L}}$$

- The amplifier has a bandwidth reduction factor of $\frac{A_{i}^{2}}{g_{m}}$

$$\omega_{0} \times |A_{i}|^{2} = \omega_{p} \times \left(\frac{R_{L}}{R_{L}}\right) \times \frac{1}{|g_{m}|}$$

Example

- Say we need a gain of 40 dB ($A_{i} = 1000$) and $\frac{g_{m}}{A_{i}} = 2$. The technology has a capacitance ratio of $\mu = 0.2$.

$$\omega_{0} \frac{|A_{i}|^{2}}{1000} = \frac{1}{\mu^{-1}} \times 2 \times 5$$

$$\omega_{0} = \frac{\omega_{p}}{1000}$$

- Compare this to a current mirror amplifier. When we follow the “normal” gain-bandwidth tradeoff, we have

$$\omega_{0} = \frac{\omega_{p}}{1000}$$

Common Base Amplifier

- Write KCL at base node of circuit

$$\frac{v_{o}+v_{i}}{R_{s}} + g_{m}v_{in} + sC_{in}v_{oa} = 0$$

$$v_{s} = -v_{ia}(1 + g_{m}R_{s} + sC_{in}R_{s})$$
Common Base Amplifier (2)

- And write KCL at the output node
  \[ (sC_v + \frac{1}{R_L})v_o + g_{m_v}v_m + sC_v v_o = 0 \]
  \[ v_o \left( \frac{1}{R_L} + s(C_v + C_p) \right) = -g_{m_v}v_m \]
- The voltage gain is a product of two terms
  \[ A_v = \frac{v_o}{v_i} = \frac{-g_{m_v}R_L}{1 + s(C_v + C_p)R_L} \frac{v_m}{v_i} \]
  \[ A_v = \frac{g_{m_v}R_L}{1 + s(C_v + C_p)R_L |1 + sR_m|} \frac{v_m}{v_i} \]

Common Base Bandwidth

- Note the transconductance is degenerated, \( G_m = g_m/(1 + g_m R_L) \). Note that the input capacitance is also degenerated by the action of series feedback.
- Unlike a CE/CS amplifier, the poles do not interact (due to absence of feedback capacitor)
- First let’s take the limit of high loop gain, \( g_{m_v} R_L \gg 1 \)
  \[ A_v = \frac{g_m}{(1 + s/\omega_L)(1 + s/\omega_L^*)} \]
  where \( \omega_L = ((C_v + C_p)R_L)^{-1} \) is the pole at the output.

Matched Common Base Amplifier

- The common-base amplifier has the nice property that the input impedance is low (roughly \( 1/g_{m_v} \)) and broadband, thus easily providing a termination to the driver (a filter, the antenna, or a previous stage). If we assume that \( R_L = 1/g_{m_v} \), we have
  \[ A_v = \frac{g_m R_L}{(1 + s/\omega_L)(1 + s/\omega_L^*)} \]
- The 3dB bandwidth is thus most likely set by the time constant at the load.

Shunt Feedback Amplifier

- The shunt-feedback amplifier is a nice high-frequency broadband amplifier building block. The action of the shunt feedback is used to lower the input impedance and to set the gain.

Gain and Input Resistance

- The in-band voltage gain and input impedance is given by (see homework)
  \[ A_v = \frac{-g_{m_v} R_L}{R_i} \]
  \[ R_{in} = (1 + \frac{R_F}{R_L}) \frac{1}{g_{m_v}} \]
- For an input match, \( R_s = (1 + \frac{R_F}{R_L})^{-1} \) or \( g_{m_v} R_s = (1 + \frac{R_F}{R_L})^{-1} \)
- Since the voltage gain sets \( R_F \), the input impedance match determines the required transconductance \( g_{m_v} \) (and hence the power dissipation)
- A bipolar version will dissipate much less power due to the higher intrinsic \( g_{m_v} \)

Shunt Feedback Amplifier BW

- The amplifier is broadband and approximately obeys the classic gain-bandwidth tradeoff \( A_v \omega_B = \omega_L \)
- A zero-value time constant analysis identifies the dominant pole
  \[ \tau_1 = C_m \left( \frac{R_s |v_s|}{R_l (1 + \frac{R_F}{R_L})} \right) \]
  \[ \tau_2 = C_p \left( \frac{R_s |v_s|}{R_l |v_s| + \frac{R_F}{R_L}} \right) \]
  \[ \omega_{-3DB} \approx \frac{1}{\tau_1 + \tau_2} \]
**Shunt Feedback/Common Collector Cascade**

- If the shunt-FB amplifier needs to drive a low impedance load, a broadband voltage buffer is needed.
- As shown below, an emitter follower (or source follower) provides the solution (note this is a fast pnp). Note the buffer is broadband (gain ≈ 1) and only loads the core amplifier by the degenerated input capacitance \(C_{GS}/(1 + \beta_{MAX}R_L)\).

**Wideband Two Stage Amplifiers**

- A source follower driving a common-source amplifier buffers the high source impedance and drives the common source amplifier with a low source impedance.
- A source follower driving a common gate amplifier boosts the input impedance. This is essentially a differential pair driven single-endedly.
- A common-source amplifier drives a common-gate amplifier, or a cascode amplifier. Miller effect is minimized by lowering the gain of the common-source stage.