MOS Voltage Switching Mixer

- A practical implementation uses MOS devices as switches. The devices are large to minimize their on-resistance with a limit determined by isolation (feed-through capacitance).

- We see that the RF signal is effectively multiplied by ±1 with a rate determined by the LO signal.

- A differential RF signal is created using a balun or fed directly from a balanced LNA.

→ REQ. SWING IS LARGER

→ LINEAR

→ NO DC CURRENT

→ NO FLICKER NOISE
MOS Switching (Passive) Mixer Summary

- MOS passive mixer is very linear. The device is either “on” or “off” and does not impact the linearity too much. Since there is no transconductance stage, the linearity is very good.
- The downside is that the MOS mixer is passive, or lossy. There is no power gain in the device.
- Need large LO drive to turn devices on/off.
- Need to create a differential RF and LO signal. This can be done using baluns or by using a differential LNA and LO buffer.
The RF/LO/IF are all differential signals. During the positive LO cycle, the RF is coupled to the IF port with positive phase, whereas during the negative phase the RF is inverted at the IF.

The MOS resistance forms a voltage divider with the source and load and attenuates the signal as before.
- Since gates of the MOS switches present a large capacitive load, a buffer is needed to drive them.
- The LO buffer can be realized using larger inverters (approach "square wave") or as a tuned buffer. A tuned lowers the power by roughly $\frac{1}{7}$ but has a sinusoidal waveform.
LO Power (Inverter Chain)

- For an inverter chain driving the LO port, the power dissipation of the last stage is given by
  \[ P_{\text{inv}} = CV_{\text{LO}}^2f_{\text{LO}} \]

- \( C \) is the total load presented to the LO (two MOS devices for the double balanced mixer, one MOS device for single balanced).

- \( V_{\text{LO}} \) is the LO amplitude to fully turn the devices on and off. The devices should be biased near threshold. \( f_{\text{LO}} \) is the LO frequency.
Tuned LO Power

- For the tuned load case, the power is given by
  \[ P_{\text{tuned}} = \frac{V_{LO}^2}{2R_t} \]

- \( R_t \) is the effective shunt resistance of the tank. Since the tank
  \( Q = \omega_LO R_t C \), we have
  \[ P_{\text{tuned}} = \frac{V_{LO}^2}{2Q} \omega_LO C = \frac{\pi CV_{LO}^2 f_LO}{Q} \]

- A high \( Q \) tank helps to reduce the power consumption of the LO
  buffer.

\[ Q = \frac{P_t}{\omega_0 L} = \frac{k_t \omega_0 C}{R_p} \]

\[ R_p = Q \frac{R_t}{\omega_0 L} \]

\[ P_{\text{tuned}} = \frac{\pi CV_{LO}^2 f_LO}{Q} \sim CV_{LO}^2 f_LO \]

\( Q \gg \pi \)

1. Efficiency: \( \eta = \frac{P_{\text{out}}}{P_{\text{in}}} \)

2. \( f \approx 10 \text{GHz} \) (65nm /45nm)
   Active/Passive
   \( C_{\text{cap}} \approx 100-200 fF \)
   \( L_{\text{reg}} = 8 \text{nH} ! \approx 10 \text{GHz} \)

\[ L_{\text{reg}} = \frac{1}{200 fF} \]

\[ C_{\text{add}} \]

\[ Q \approx 5-6 \]
If PMOS devices are available, two CMOS inverters form an H-Bridge, applying the RF input signal to the IF directly during the LO cycle and inverting the RF input at the IF output in the LO cycle.

PMOS and NMOS devices are sized appropriately to maximize on-conductance and to minimize off capacitance.
Time-Varying Conductance

- The RF voltage is applied to a time varying conductance. Note that if the conductance of the LO switches is given by \( g(t) \), then the conductance of the LO switches is given by \( g(t - T_{LO}/2) \).

The Thevenin equivalent source voltage is given by the open circuit voltage

\[
\begin{align*}
\nu_T &= v_{RF} \left( \frac{g(t)}{g(t) + g(t - T_{LO}/2)} \right) - \frac{g(t - T_{LO}/2)}{g(t) + g(t - T_{LO}/2)} \right) \\
v_T &= v_{RF} \left( \frac{g(t)}{g(t) + g(t - T_{LO}/2)} \right) = m(t)v_{RF}
\end{align*}
\]

\( g(t) \)
\( g(t + T) = g(t) \)
\( g_{on} \)
\( g_{off} \)

\( v_T = v_{RF} (m(t)) \)

Junc Caps \( \xrightarrow{\text{nonlinear}} \) Distortion.
Time-Varying Gain $m(t)$

- For the MOS device and a given LO waveform, the function $m(t)$ can be calculated and the Fourier expansion can be used to derive the gain.
- In practice there is a load capacitance $C_{IF}$ at the IF port to filter the downconverted signal. This $C_{IF}$ complicates the analysis but interested students are encouraged to read the paper by Shahani, Shaefler and Lee (JSSC vol. 32, Dec 1997, p. 2061-1071).
Note that the Gilbert quad is really a folded ring. Thus the passive and active mixers are very similar. The main difference is how the quad devices are biased. In the Gilbert cell they are biased nominally in saturation and have DC current. In the passive mixers, they are biased near the threshold.
APPENDIX I:
PERIODIC STEADY STATE
SIMULATIONS
Also refer to Discussion notes of 11/08/2010
Periodic Steady State (PSS) Simulations

- Transient simulation is slow and costly because we have to do a tight tolerance simulation of several IF cycles with a weak RF.
- The SpectreRF PSS analysis is a tool for finding the periodic steady-state solution to a circuit. In essence, it tries to find the initial condition or state for the circuit (capacitor voltages, inductor currents) such that the circuit is in periodic steady state.
- It can usually find the periodic solution within 4-5 iterations.
- In the mixer, if we ignore the RF signal, then the periodic operating point is determined by the LO signal alone.
PSS Iteration

- Since typical PSS run converges in 4-5 cycles of the LO, or a simulation time of about $5T_{LO}$, the overall simulation converges several orders of magnitude faster than transient at the IF frequency.
- PSS requires that the circuit is not chaotic (periodic input leads to a periodic output).
- High $Q$ circuits do not pose a problem to PSS simulation since we are finding the steady-state solution. The high $Q$ natural response takes roughly $Q$ cycles to die down, thus saving much simulation time.
PSS Options

- We can perform PSS analysis on driven or autonomous circuits. An autonomous circuit has no periodic inputs but produces a periodic output (e.g., an oscillator).
- For PSS analysis, we need to specify a list of "large" signals in the circuit. In a mixer, the only large tone is the LO, so there is only one signal to list.
- We also specify the "beat frequency" or the frequency of the resulting periodic operating point. For instance, if we drive a circuit with two large tones at $f_1$ and $f_2$, the beat frequency is $|f_1 - f_2|$. Spectre can auto-calculate this frequency.
- An additional time for stabilization $t_{stab}$ can be specified to help with the convergence. For a mixer, this is not needed.
Periodic AC (PAC) Simulations

- Once a PSS analysis is performed at the “beat frequency”, the circuit can be linearized about this time varying operating point.
- Note that for a given AC input, there are as many transfer functions as there are harmonics in the LO.
- We specify the frequency range of the AC input signal as either an absolute or relative range. A relative range is a frequency offset with respect to the beat frequency.
- We also specify the maximum sidebands to keep for the simulation. Note that this does not affect the accuracy of the simulation but simply the amount of saved data.
The input frequency $f_i$ is translated to frequencies $f_i + kf_o$, where $f_i$ is the beat (LO) frequency. The $k = 0$ sideband corresponds to the DC component of the LO signal (e.g. time invariant behavior). The non-zero components, though, correspond to mixing. E.g. $k = 1$ correspond to frequency down-conversion. $k = +1$ is the normal up-conversion. $k = -2$ is the 2nd harmonic mixing $f_i - 2f_o$. 
PNoise is a noise analysis that takes the frequency translation effects into account. The simulation parameters are similar to PAC with the exception that we must identify the input and output ports (for noise figure) and the reference side-band, or the desired output frequency. For a mixer, this is $k = -1$. 
OSCILLATORS
**Oscillators**

- An oscillator is an autonomous circuit that converts DC power into a periodic waveform. We will initially restrict our attention to a class of oscillators that generate a sinusoidal waveform.
- The period of oscillation is determined by a high-Q L.C tank or a resonator (crystal, cavity, T-line, etc.). An oscillator is characterized by its oscillation amplitude (or power), frequency, "stability", phase noise, and tuning range.

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- DC
- AC
- Freq. Stab
- Amp. Stab
- Power Consumption
- Tuning Range

VCO
Voltage-Cont. Osc. Current
Generically, a good oscillator is stable in that its frequency and amplitude of oscillation do not vary appreciably with temperature, process, power supply, and external disturbances.

- The amplitude of oscillation is particularly stable, always returning to the same value (even after a disturbance).

Stability
long term, short term.
Due to noise, a real oscillator does not have a delta-function power spectrum, but rather a very sharp peak at the oscillation frequency.

The amplitude drops very quickly, though, as one moves away from the center frequency. E.g. a cell phone oscillator has a phase noise that is 100 dB down at an offset of only 0.01% from the carrier!
- Note that an LC tank alone is not a good oscillator. Due to loss, no matter how small, the amplitude of the oscillator decays.
- Even a very high $Q$ oscillator can only sustain oscillations for about $Q$ cycles. For instance, an LC tank at 1GHz has a $Q \approx 20$, can only sustain oscillations for about 20ns.
- Even a resonator with high $Q \approx 10^6$, will only sustain oscillations for about 1ns.

![Diagram of LC Tank](image)
Feedback Perspective

Many oscillators can be viewed as feedback systems. The oscillation is sustained by feeding back a fraction of the output signal, using an amplifier to gain the signal, and then injecting the energy back into the tank. The transistor "pushes" the LC tank with just about enough energy to compensate for the loss.

\[ H(\alpha) = \frac{Y}{X} = \frac{M}{1 + H} \]

\[ \frac{Y}{X} = \begin{cases} \text{finite} & \text{if } X = 0 \text{ or } \infty \end{cases} \]

\[ |H| = 1 \]
\[ \arg H = 180^\circ \]

Positive Feedback (POS FB)

Builds up (ENFORCED SELF)
Another perspective is to view the active device as a negative resistance generator. In steady state, the losses in the tank due to conductance $G$ are balanced by the power drawn from the active device through the negative conductance $-G$. 
Consider an ideal feedback system with forward gain $a(s)$ and feedback factor $f(s)$. The closed-loop transfer function is given by

$$H(s) = \frac{a(s)}{1 + a(s)f(s)}$$
Feedback Example

- As an example, consider a forward gain transfer function with three identical real negative poles with magnitude \(|\omega_p| = 1/\tau\) and a frequency independent feedback factor \(f\)
  
  \[
  a(s) = \frac{a_0}{(1 + s\tau)^3}
  \]

- Deriving the closed-loop gain, we have
  
  \[
  H(s) = \frac{a_0}{(1 + s\tau)^3 + a_0f} = \frac{K_1}{(1 - s/s_1)(1 - s/s_2)(1 - s/s_3)}
  \]

- where \(s_{1,2,3}\) are the poles of the feedback amplifier.

\[
\omega_p = \frac{1}{\tau}
\]

\[
H(s) = \frac{a(s)}{1 + a(s)f} = \frac{a_0}{(1 + s\tau)^3}
\]

\[
\frac{S_o(s)}{S_i} = H(s) = \frac{a_0}{(1 + s\tau)^3 + a_0f}
\]

\[
\frac{S_o(s)}{S_i} = \frac{K_1}{b_0 + b_1s + b_2s^2 + b_3s^3}
\]

\[
(1 + s\tau)^3 + a_0f = \frac{a_0}{(1 - s/s_1)(1 - s/s_2)(1 - s/s_3)}
\]

\[
s_1, s_2, s_3
\]
Poles of the Closed Loop System

- Solving for the poles

\[
(1 + s\tau)^3 = -a_0f
\]

\[
1 + s\tau = (-a_0f)^\frac{1}{3} = (a_0f)^\frac{1}{3}(-1)^\frac{1}{3}
\]

\[
(-1)^\frac{1}{3} = -1, \quad e^{j60^\circ}, \quad e^{-j60^\circ}
\]

- The poles are therefore

\[
s_1, s_2, s_3 = \frac{-1 - (a_0f)^\frac{1}{3}}{\tau}, \quad \frac{-1 + (a_0f)^\frac{1}{3}e^{j60^\circ}}{\tau}, \quad \frac{-1 + (a_0f)^\frac{1}{3}e^{-j60^\circ}}{\tau}
\]
**Root Locus**

- If we plot the poles on the s-plane as a function of the DC loop gain \( T_0 = a_0f \), we generate a root locus.
- For \( a_0f < 8 \), the poles are on the \( j\omega \)-axis with value:
  \[
  s_1 = -\frac{3}{\tau}
  \]
  \[
  s_{2,3} = \pm j\sqrt{3}/\tau
  \]
- For \( a_0f > 8 \), the poles move into the right-half plane (RHP).

\[
P_1 = \frac{-1 - (a_0f)^{1/3}}{2}
\]
\[
P_{2,3} = \frac{-1 + (a_0f)^{1/3} \pm j60^\circ}{2}
\]

Pole pos \( \Rightarrow \) feedback loop gain \( a_0f \)

\( a_0f = 0 \) (No FB)

\( a_0f = 8 \) (More FB)

**Natural Response**
Natural Response

- Given a transfer function
  \[ H(s) = \frac{K}{(s - s_1)(s - s_2)(s - s_3)} = \frac{a_1}{s - s_1} + \frac{a_2}{s - s_2} + \frac{a_3}{s - s_3} \]

- The total response of the system can be partitioned into the natural response and the forced response
  \[ s_0(t) = f_1(a_1e^{s_1t} + a_2e^{s_2t} + a_3e^{s_3t}) + f_2(s_i(t)) \]

  where \( f_2(s_i(t)) \) is the forced response whereas the first term \( f_1() \) is the natural response of the system, even in the absence of the input signal. The natural response is determined by the initial conditions of the system.
Real LHP Poles

- Stable systems have all poles in the left-half plane (LHP).
- Consider the natural response when the pole is on the negative real axis, such as $s_1$ for our examples.
- The response is a decaying exponential that dies away with a time-constant determined by the pole magnitude.

$$F(s) = \frac{a_0}{s - \alpha}$$

$$f(t) = e^{-\alpha t}$$
Complex-Conjugate LHP Poles

- Since $s_{2,3}$ are a complex conjugate pair
  \[ s_2, s_3 = \sigma \pm j\omega \]

- We can group these responses since $a_3 = \overline{a_2}$ into a single term
  \[ a_2 e^{\sigma t} + a_3 e^{\sigma t} = K e^{\sigma t} \cos \omega t \]

- When the real part of the complex conjugate pair $\sigma$ is negative, the response also decays exponentially.
Complex Conjugate RHP Poles

- When $\sigma$ is positive (RHP), the response is an exponential growing oscillation at a frequency determined by the imaginary part $\omega_0$.
- Thus we see for any amplifier with three identical poles, if feedback is applied with loop gain $T_0 = a_0 f > 8$, the amplifier will oscillate.

$a_0f > 8 \rightarrow$ osc. !

→ STABLE AMP STARTUP

LINEAR ANALYSIS

- TRANS. REGION
- TIME VARIANCE
  + NON-LIN.

STATIONARY

5V!
In the frequency domain perspective, we see that a feedback amplifier has a transfer function

\[ H(j\omega) = \frac{a(j\omega)}{1 + a(j\omega)f} \]

- If the loop gain \( a_0f = 8 \), then we have with purely imaginary poles at a frequency \( \omega_c = \sqrt{3}/f \) where the transfer function \( a(j\omega)f = -1 \) blows up. Apparently, the feedback amplifier has infinite gain at this frequency.

**Forced Response:** \( w < w_0 \)

\[ w \times \text{comp} \times \text{TF} \times w \]

\[ \frac{S_i}{S_i} \rightarrow \infty \]

\[ S_i \rightarrow \text{still have } S_0 \]
In a real oscillator, the amplitude of oscillation initially grows exponentially as our linear system theory predicts. This is expected since the oscillator amplitude is initially very small and such theory is applicable. But as the oscillations become more vigorous, the non-linearity of the system comes into play.

We will analyze the steady-state behavior, where the system is non-linear but periodically time-varying.

\[ \text{Osc} \rightarrow \text{Amplitude} \rightarrow \text{x} \]
LC Oscillator

- The emitter resistor is bypassed by a large capacitor at AC frequencies.
- The base of the transistor is conveniently biased through the transformer windings.

The LC oscillator uses a transformer for feedback. Since the amplifier has a phase shift of 180°, the feedback transformer needs to provide an additional phase shift of 180° to provide positive feedback.

- $R_W = \frac{\omega_0}{Q}$
- $Q \uparrow \Rightarrow BW \downarrow$ sharper filter / phase transition steeper.
- Phase noise $\propto \frac{1}{Q^2}$
At resonance, the AC equivalent circuit can be simplified. The transformer winding inductance $L$ resonates with the total capacitance in the circuit $C_T$ is the equivalent tank impedance at resonance.

\[
\frac{V_o}{V_i} \quad \text{Feed Forward} \rightarrow -j\omega L \quad (180°)
\]

\[
FB \rightarrow \frac{V_i}{V_o} = \frac{1}{n} \quad (180°)
\]

Total phase = 0°
The forward gain is given by $a(s) = -g_m Z_T(s)$ where the tank impedance $Z_T$ includes the loading effects from the input of the transistor.

\[
R = R_0 || R_i || n^2 R_i
\]

\[
C = C_L + \frac{C_i}{n^2}
\]

where $f = V_{in}$

\[
R = R_0 || R_i || n^2 R_i
\]
Open Loop Transfer Function

- The tank impedance is therefore
  \[ Z_T(s) = \frac{1}{sC + \frac{1}{R} + \frac{1}{Ls}} = \frac{Ls}{1 + s^2LC + sL/R} \]

- The loop gain is given by
  \[ a_f(s) = \frac{-g_mR}{n} \frac{\frac{1}{R}s}{1 + \frac{1}{R}s + s^2LC} \]

- The loop gain at resonance is the same as the DC loop gain
  \[ A_L = \frac{-g_mR}{n} \frac{|A_L| = \frac{g_mR}{n}} \]

\[ a_f = (g_mR) \times \left(\frac{1}{n}\right) \]
Closed Loop Transfer Function

- The closed-loop transfer function is given by

\[ H(s) = \frac{-g_m R \frac{L}{R} s}{1 + s^2 LC + s \frac{L}{R} (1 - \frac{g_m R}{L})} \]

- Where the denominator can be written as a function of \( A_t \)

\[ H(s) = \frac{-g_m R \frac{L}{R} s}{1 + s^2 LC + s \frac{L}{R} (1 - A_t)} \]

- Note that as \( n \to \infty \), the feedback loop is broken and we have a tuned amplifier. The pole locations are determined by the tank \( Q \).
If $A_f = 1$, then the denominator loss term cancels out and we have two complex conjugate imaginary axis poles:

$$1 + s^2 LC = (1 + sj\sqrt{LC})(1 - sj\sqrt{LC})$$
Magnitude of the Poles

For a second order transfer function, notice that the magnitude of the poles is constant, so they lie on a circle in the s-plane

\[ s_{1,2} = \frac{-a}{2b} \pm \frac{a}{2b} \sqrt{1 - \frac{4b}{a^2}} = \frac{-a}{2b} \pm j \frac{a}{2b} \sqrt{\frac{4b}{a^2} - 1} \]

\[ |s_{1,2}| = \sqrt{\frac{a^2}{4b^2} + \frac{a^2}{4b^2} \left(\frac{4b}{a^2} + 1\right)} = \sqrt{\frac{1}{b}} = \omega_0 \]
We see that for $A_L = 0$, the poles are determined by the tank Q and lie in the LHP. As $A_L$ is increased, the action of the positive feedback is to boost the gain of the amplifier and to decrease the bandwidth. Eventually, as $A_L = 1$, the loop gain becomes infinite in magnitude.