Electronic Noise

- Why is noise important?
  - Sets minimum signals we can deal with – often sets lower limit on power

- Signal-to-noise ratio
  - Signal Power \( P_{\text{sig}} \approx (V_{DD})^2 \)
  - Noise Power \( P_{\text{noise}} = kT/C \)
  - SNR = \( P_{\text{sig}} / P_{\text{noise}} \)

- Technology Scaling
  - \( V_{DD} \) goes down \( \rightarrow \) lower signal
  - Increase \( C \) to compensate \( \rightarrow \) increases power

Types of “Noise”

- Interference
  - “fundamental” – deterministic
  - Signal coupling
    - Capacitive, inductive, substrate, etc.
  - Supply noise

- Device noise
  - Caused by discreteness of charge
  - “fundamental” – thermal noise
  - “manufacturing process related” – flicker noise

Resistor Noise Model

- Origin: Brownian Motion
  - Thermally agitated particles
  - E.g. ink in water, electrons in a conductor

- Available noise power:
  - \( P_{n} = kT\Delta f \)
  - Noise power in bandwidth \( \Delta f \) delivered to a matched load
  - Example: \( \Delta f = 1 \text{Hz} \rightarrow P_{n} = 4 \times 10^{-21} \text{W} = -174 \text{ dBm} \)

Resistor Noise Model

- Mean square noise voltage:
  - \( \overline{v_n^2} = 4kT\Delta f \)
Thermal Noise

- Present in all dissipative elements
  - i.e., resistors
- Independent of DC current flow
- Random fluctuations of $v(t)$ or $i(t)$
  - Mean is 0
  - Distribution (pdf) is Gaussian
  - Power spectral density is “white”
  - Up to $\sim$THz frequencies
  - $k_B T = 4 \times 10^{-21} J \quad (T = 290K = 16.9^{\circ}C)$
- Example:
  $R = 1k \Omega \Rightarrow 4nV/\sqrt{Hz}$
  $1MHz$ bandwidth $\Rightarrow \sigma = 4uV$

Noise of Passive Networks

- Capacitors and inductors only shape spectrum
- Noise calculations
  - Instantaneous voltages add
  - Power spectral densities add
  - RMS voltages do NOT add
- Example: $R_1 + R_2$ in series
- Generalization to arbitrary RLC networks

Shot noise in Diodes

- Zero mean, Gaussian pdf, white
- Independent of temperature
- Example:
  $I_s = 1mA \Rightarrow 17.9pA/\sqrt{Hz}$
  $1MHz$ bandwidth $\Rightarrow \sigma = 17.9nA$

BJT Noise

- Just like diodes: shot noise
  - Collector and base noise partially correlated
- Extrinsic resistors contribute noise
  - Small signal resistors (e.g., $r_h$) don’t
  - These aren’t physical resistors

FET Noise

- Channel resistance contributes thermal noise
- Channel conductance:
  $$g_{soa} = \frac{W}{L}(V_{OA} - V_{GS}) = g_m$$
- Noise injection is actually distributed across the channel (note $\gamma$):
  $$\overline{V_i^2} = 4kT \gamma g_m \Delta f$$

More Fundamental Expression

- More fundamental equation uses channel charge
  [Tsividis]
- When $V_{ds} = 0$, device is truly a resistor:
  $$\overline{I_{ds}^2} = 4kT \frac{W}{L^2} C_{ox} (V_{GS} - V_T) \Delta f$$
**Strong Inversion Noise**

- In saturation, drain current noise is
  \[ \overline{I_d^2} = 4kT \frac{2W}{3L} \mu C_{ox} (V_{GS} - V_T) \Delta f \]

- For long channel model, can substitute \( g_m \) for the above factor.

- In practice, form involving actual inversion charge is more accurate
  - This is what SPICE/BSIM use

**Weak Inversion**

- Weak inversion: BJT \( \rightarrow \) shot noise.
  - Result should be \( \sim 2g_{ds} \)

- Get the same result from inversion charge expression:
  \[ Q_i = W_i \frac{Q_{id}}{2} + Q_{id} = \frac{L^2}{2\mu C_{ox}} \frac{q V_{DS}}{q} \left( 1 + e^{-q V_{DS}/kT} \right) \]

- \( \overline{I_d^2} = 2q I_{DS} \left( 1 + e^{-q V_{DS}/kT} \right) \Delta f \)

**FET Noise Model**

- Model neglects intrinsic gate noise
- BSIM3 does not directly include \( \alpha \)

**Thermal Noise for Short Channels**

- Strong inversion \( \rightarrow \) thermal noise
  - Drain current: \( g_{mot} \) is what you really care about
    \[ \overline{I_d^2} = 4kT \gamma g_{mot} \Delta f = 4kT \gamma g_m \Delta f \]
  - \( g_m \) more convenient for input-referred noise
    - For low field (long \( L \)), \( \gamma = 2/3 \) relates \( g_m \) to \( g_{ds} \)
    - For high field, use \( \alpha \) to capture increase in noise
  - High-field noise can be 2-3 times larger than low field

- MOS actually has intrinsic gate induced noise (142/244 topic)
- Gate leakage \( \rightarrow \) shot noise

**1/f Noise**

- Flicker noise
  - \( K_{NMOS} = 2.0 \times 10^{-29} \text{ AF} \)
  - \( K_{PMOS} = 3.5 \times 10^{-30} \text{ AF} \)
  - Strongly process dependent
  \[ \overline{I_{f/ff}} = \frac{K_{fD} I_{D}}{L^2 C_{ox}} \Delta f \]

- Example: \( I_D = 10 \mu A, \ L = 1 \mu m, \ C_{ox} = 5.3 \text{fF/}\mu m^2, \ f_L = 1 \text{MHz} \)
  - \( f_L = 1 \text{Hz} \) \( \Rightarrow \sigma = 722 \text{pA} \)
  - \( f_L = 1 \text{year} \) \( \Rightarrow \sigma = 1083 \text{pA} \)
  \[ \overline{I_{f/ff,\text{total}}} = \overline{I_{f10}} + \frac{K_{fD} I_{D} \Delta f}{L^2 C_{ox} f} = \frac{K_{fD} I_{D} \ln f_L}{L^2 C_{ox} f} \]

**1/f Noise Corner Frequency**

- Definition (MOS)
  \[ K_{fD} \frac{1}{f_{ce}} = 4kT \gamma g_{mot} \]
  \[ f_{ce} = \frac{K_{fD}}{4kT \gamma g_{mot}} = \frac{K_f}{8kT \gamma C_{ox}} \]

- Example:
  - \( V^* = 200 \text{mV}, \ \gamma = 1 \)
  - \( L = 0.35 \mu m \) \( \Rightarrow \) NMOS 192kHz PMOS 34kHz
  - \( L = 1.00 \mu m \) \( \Rightarrow \) 24kHz 4kHz
Noise Calculations

- Method:
  1) Create small-signal model
  2) All inputs = 0 (linear superposition)
  3) Pick output $v_o$ or $i_o$
  4) For each noise source $v_x, i_x$
     Calculate $H_x(s) = v_o(s) / v_x(s)$ ($... i_o, i_x$)
  5) Total noise at output is:
     $$v_{ox}^2(f) = \sum_{x} H_x(s) \int_{-\infty}^{\infty} v_x(f)$$
     simpler notation: $v_{ox}^2(f) = S_v(f)$

Tedious but simple …

Example: Common Source