Noise Variance in a Real Circuit: Sample and Hold

- Noise on the capacitor:
  \[ v_{on}^2(f) = 4k_B T R \left( \frac{1}{1 + sRC} \right)^2 \]
  \[ \Rightarrow \overline{v_{oT}^2} = \int_{0}^{\infty} v_{on}^2(f) df = \frac{k_B T}{C} \]

- So effective bandwidth is:
  \[ 4k_B T R \Delta f = \frac{k_B T}{C} \]
  \[ \Rightarrow \Delta f = \frac{1}{4RC} = \frac{\pi}{2} f_o \]
SPICE Verification

Energy-Based Analysis
Useful Integrals

\[ \int_0^\infty \frac{1}{1 + \frac{s}{\omega_0}} \, df = \frac{\omega_0}{4} \]

\[ \int_0^\infty \frac{1}{1 + \frac{s}{\omega_0 Q} + \frac{s^2}{\omega_0^2}} \, df = \frac{\omega_0 Q}{4} \]

\[ \int_0^\infty \frac{s}{\omega_0} + 1 \frac{1}{1 + \frac{s}{\omega_0 Q} + \frac{s^2}{\omega_0^2}} \, df = \frac{\omega_0 Q}{4} \left( \frac{\omega_0^2}{\omega_0^2 + 1} \right) \]

CS Amplifier

\[ v_{in}(f) = 4k_d T \left( \frac{1}{R_L} + \frac{2}{3} g_m \right) R_L \left[ \frac{1}{1 + s R_L C_L} \right]^e \]

\[ v_{out} = 4k_d T \left( \frac{1}{R_L} + \frac{2}{3} g_m \right) R_L^2 \frac{1}{4 R_L C_L} \frac{1}{1 + s R_L C_L} \]

\[ = 4k_d T \left( \frac{1}{R_L} + \frac{2}{3} g_m R_L \right) \frac{1}{4 R_L C_L} \]

\[ = \frac{k_T}{C_L} \left( 1 + \frac{2}{3} g_m R_L \right) \]

\[ = \frac{k_T}{C_L} \left( 1 + \frac{2}{3} |A_m| \right) \]

\[ = \frac{k_T}{C_L} R_T \]

\[ = \frac{V_{in}}{V_{out}} \]
Signal-To-Noise Ratio

- **SNR:**
  \[ SNR = \frac{P_{\text{sig}}}{P_{\text{noise}}} \]

- **Signal Power (sinusoidal source):**
  \[ P_{\text{sig}} = \frac{1}{2} V^2 \text{zero–peak} \]

- **Noise Power (assuming thermal noise dominates):**
  \[ P_{\text{noise}} = \frac{k_b T}{C} n_f \]

- **So:**
  \[
  SNR = \frac{\frac{1}{2} CV^2_{\text{zero–peak}}}{n_f k_b T} \]

- **SNR** \( \uparrow \) +6dB
  \[ C \downarrow \times 4 \]

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dB versus Bits

- **Quantization “noise”**
  - **Quantizer step size:** \( \Delta \)
  - **Box-car pdf variance:** \( S_Q = \frac{\Delta^2}{12} \)

- **SNR of N-Bit sinusoidal signal**
  - **Signal power**
    \[ P_{\text{sig}} = \frac{1}{2} \left( 2^N \frac{\Delta}{2} \right)^2 \]
  - **SNR**
    \[ SNR = \frac{P_{\text{sig}}}{S_Q} = 1.5 \times 2^N \]
  - **6.02 dB per Bit**
    \[ = \left( 1.76 + 6.02 N \right) \text{ dB} \]
SNR versus Power

- 1 Bit → 6dB → 4x SNR
- 4x SNR → 4x C
- Circuit bandwidth \( \sim g_m/C \) → 4x \( g_m \)
- Keeping \( V^* \) constant → 4x \( I_D \), 4x W

- Thermal noise limited circuit:
  - Each bit QUADRUPLES power!

- Overdesign is expensive
  - Better do the analysis right!

Analog Circuit Dynamic Range

- Biggest signal set by \( V_{DD} \). So, for (single-ended) sinusoid:
  \[
  V_{\text{max}}(\text{rms}) = \frac{1}{\sqrt{2}} \frac{V_{DD}}{2}
  \]

- The noise is
  \[
  V_n(\text{rms}) = \sqrt{n_f \frac{k_B T}{C}}
  \]

- So the dynamic range in dB is:
  \[
  DR = \frac{V_{\text{max}}(\text{rms})}{V_n(\text{rms})} = \frac{V_{DD} \sqrt{C}}{\sqrt{8n_f k_B T}} \quad [V/V]
  \]
  \[
  = 20 \log_{10} \left( V_{DD} \sqrt{\frac{C}{n_f}} \right) + 75 \quad [\text{dB}] \text{ with } C \text{ in [pF]}
  \]
Analog Circuit Dynamic Range

- Biggest swing set by supply voltage $V_{DD}$
  \[ SNR = \frac{1}{2} \frac{CV_{DD}^2}{n_f k_b T} \]

- Modern ICs: $V_{DD} \approx 1V$, $C < \approx 1nF$ ($n_f = 1$)
  - DR < 100dB (~16 Bits)

- PCB circuits with 30V and discrete $C$ of ~100nF:
  - DR < 140dB (23 Bits)
  - A 40dB (~7 Bit) advantage!

- Note: can break this barrier with oversampling

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Sampled Noise Spectrum

\[ S_n(f) = \frac{k_b T_c}{C} \frac{2}{f_s} \frac{1 - e^{-2\pi f f_c}}{1 + e^{-2\pi f f_c}(1 - \cos 2\pi f T)} \]
\[ a = \frac{T}{R_c C} = \frac{T}{\tau} \quad \text{and} \quad T = \frac{1}{f_s} \]
\[ \int_0^\infty S_n(f) df = \frac{k_b T_c}{C} \]

- What if RC doesn’t completely settle every cycle?
  - Noise between samples correlated → spectrum not white
  - If $T/\tau > 3$, correlation small
    - Sampled spectrum white
    - In practice usually the case
Periodic Noise Analysis

SpectreRF PNOISE: check noisetype=timedomain noisetimepoints= as alternative to ZOH. noiseskipcount=large might speed up things in this case.

Two-Stage Amplifier
Input Equivalent Noise

Equivalent Noise Generators

- Model for noisy two-port:
  - *Noiseless* two-port
  - Plus equivalent input noise sources

- In general, $v_n$ and $i_n$ are correlated.
  - Ignore that for now
Finding the Equivalent Generators

- Find $v_n$ and $i_n$ by opening and shorting the input

  - **Shorted input:**
    - Output noise due only to $v_n$
  - **Open input:**
    - Output noise due only to $i_n$

Role of Source Resistance

- If $R_s$ is large:
  - Design amplifier with low $i_n$ (MOS)
- If $R_s$ is low:
  - Design amplifier with low $v_n$ (BJT)

- For a given $R_s$, there is an optimal $v_n/i_n$ ratio
  - Alternatively, for a given amp, there is an optimal $R_s$
Total Output Noise

\[ \overline{v_n^2} = \left( \overline{v_n^2} A_s^2 + \overline{v_i^2} A_s^2 \right) \left( \frac{R_{in}}{R_{in} + R_s} \right)^2 + \left( \frac{R_{in}}{R_{in} + R_s} \right)^2 R_s^2 \overline{v_i^2} A_s^2 \]

\[ = \left( \overline{v_n^2} + \overline{v_i^2} R_s^2 + \overline{v_i^2} R_s^2 \right) \left( \frac{R_{in}}{R_{in} + R_s} \right)^2 A_s^2 \]

New Equivalent Generator

\[ \overline{v_{eq}^2} = \overline{v_n^2} + \overline{v_i^2} R_s^2 \]

- With known \( R_s \), total noise can be lumped into one \( \overline{v_{eq}^2} \)
Optimum Source Impedance

- Can use this to optimize source impedance for minimum added noise from two-port (noise figure):

\[ R_n \equiv \frac{v_n^2}{4kT\Delta f} \quad \quad G_n \equiv \frac{i_n^2}{4kT\Delta f} \]

\[ R_{opt} = \sqrt{\frac{R_n}{G_n}} = \sqrt{\frac{v_n^2}{i_n^2}} \]

Correlated Noise Sources

- Partition \( i_n \) into two components:
  - Correlated ("parallel") to \( v_n \)
  - Uncorrelated ("perpendicular") to \( v_n \)

\[ i_n = i_c + i_u \]

\[ <i_u, v_n> = 0 \]

\[ i_c = Y_C v_n \]
Correlated Noise Sources (cont.)

Finding $Y_c$:

\[ \bar{v}_{eq}^2 = \bar{v}_n^2 + Z_s^2 \bar{i}_n^2 \]
\[ = \bar{v}_n^2 + Z_s^2 (\bar{i}_c + \bar{i}_u)^2 \]
\[ = \bar{v}_n^2 + Z_s^2 \bar{i}_c^2 + Z_s^2 \bar{i}_u^2 \]
\[ = \bar{v}_n^2 (1 + Z_s^2 Y_c^2) + Z_s^2 \bar{i}_u^2 \]
\[ \bar{v}_n^2 = \alpha_1^2 \bar{v}_1^2 + \alpha_2^2 \bar{i}_2^2 \]
\[ \bar{i}_n^2 = \beta_1^2 \bar{i}_1^2 + \beta_2^2 \bar{v}_3^2 \]
\[ \bar{i}_c^2 = \frac{i_c^2}{v_n^2} \]
\[ \bar{i}_u^2 = \frac{\beta_2^2 \bar{i}_2^2}{\alpha_1^2 \bar{v}_1^2 + \alpha_2^2 \bar{i}_2^2} \]

Equivalent Noise Voltage (cor)

- Since the above expression is the sum of two uncorrelated noise voltages, we have

\[ \bar{v}_{eq}^2 = \bar{v}_n^2 \left| 1 + Y_C Z_S \right|^2 + \left| Z_S \right|^2 \bar{i}_u^2 \]

- Now we can continue as before to find

\[ B_{opt} = B_s = -B_c \]
\[ G_{opt} = G_s = \sqrt{\frac{G_u}{R_n} + G_c^2} \]