Noise Variance in a Real Circuit: Sample and Hold

- Noise on the capacitor:
  \[ v_n^2(f) = 4k_T R C f \]
  \[ \Rightarrow v_n^2(f) = \frac{1}{B_0} \]

- So effective bandwidth is:
  \[ \Delta f = \frac{1}{4RC} \]
  \[ \Rightarrow \Delta f = \frac{1}{2B_0 f} \]

Useful Integrals

\[ \int \frac{1}{1+x^2} \, dx = \tan^{-1}x + C \]

\[ \int \frac{1}{x^2 + a^2} \, dx = \frac{1}{a} \tan^{-1} \left( \frac{x}{a} \right) + C \]

\[ \int \frac{x}{x^2 + a^2} \, dx = \frac{1}{2a} \ln(x^2 + a^2) + C \]

CS Amplifier

\[ v_{in}^2 = 4k_T R C f \]

\[ v_{out}^2 = 4k_T R C f \]

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Signal-To-Noise Ratio

- **SNR:**
  \[ SNR = \frac{P_{sig}}{P_{noise}} \]

- **Signal Power (sinusoidal source):**
  \[ P_{sig} = \frac{1}{2}V^2_{peak} \]

- **Noise Power (assuming thermal noise dominates):**
  \[ P_{noise} = \frac{kT}{C}n_i \]

So:
\[ SNR = \frac{4CV^2_{peak,noise}}{kTn_i} \]

\[ \text{SNR } \uparrow +6\text{dB} \]

\[ C \uparrow \times 4 \]

***dB versus Bits***

- **Quantization “noise”**
  \[ \Delta \]

- **Quantizer step size:**
  \[ \Delta = \frac{2^N - 1}{2^N} \]

- **Box-car pdf variance:**
  \[ S_0 = \frac{N^2}{2} \]

- **SNR of N-Bit sinusoidal signal**
  \[ \text{SNR} = \frac{P_{sig}}{P_{noise}} = 1.5 \times 2^N \]

- **6.02 dB per Bit**
  \[ = [7.76 + 6.02N] \text{ dB} \]

***SNR versus Power***

- **1 Bit \( \rightarrow 6\text{dB} \rightarrow 4x \text{SNR}**
- **4x SNR \( \rightarrow 4x \text{C}**
- **Circuit bandwidth \( \sim g_m/C \rightarrow 4x \text{ g}_m**
- **Keeping \( V^* \) constant \( \rightarrow 4x \text{ I}_o, 4x \text{ W}**

- **Thermal noise limited circuit:**
  - Each bit QUADRUPLES power!

- **Overdesign is expensive**
  - Better do the analysis right!

***Analog Circuit Dynamic Range***

- **Biggest signal set by \( V_{DD} \). So, for (single-ended) sinusoid:**
  \[ V_{max}(\text{rms}) = \frac{1}{\sqrt{2}} \frac{V_{DD}}{2} \]

- **The noise is**
  \[ F_n(\text{rms}) = \sqrt{\frac{kT}{C}} \]

So the dynamic range in dB is:
\[ DR = 20 \log \left( \frac{V_{max}(\text{rms})}{F_n(\text{rms})} \right) \text{[V/V]} \]

\[ = 20 \log \left( \frac{V_{DD}}{n_i} \right) + 75 \text{ [dB]} \text{ with C in } [\text{pF}] \]

***Analog Circuit Dynamic Range***

- **Biggest swing set by supply voltage \( V_{DD} \):**
  \[ SNR = \frac{4CV^2_{peak}}{n_i kT} \]

- **Modern ICs:** \( V_{DD} \sim 1\text{V}, C \sim 1\text{nF} \) \( n_i = 1 \)
  - DR \( > 100\text{dB} \) \( \sim 16 \text{ Bits} \)

- **PCB circuits with 30V and discrete C of \( \sim 100\text{nF}:**
  - DR \( < 140\text{dB} \) \( \sim 23 \text{ Bits} \)
  - A 40dB \( \sim 7 \text{ Bit} \) advantage!

**Note:** can break this barrier with oversampling

***Sampled Noise Spectrum***

- **What if RC doesn’t completely settle every cycle?**
  - Noise between samples correlates \( \rightarrow \) spectrum not white

- If \( T/\tau > 3 \), correlation small
- Sampled spectrum white
  - In practice usually the case
Periodic Noise Analysis

Two-Stage Amplifier

Input Equivalent Noise

Equivalent Noise Generators

Finding the Equivalent Generators

Role of Source Resistance

• Find $v_n$ and $i_n$ by opening and shorting the input
• Shorted input:
  • Output noise due only to $v_n$
  • Open input:
  • Output noise due only to $i_n$

• Model for noisy two-port:
  • Noiseless two-port
  • Plus equivalent input noise sources
• In general, $v_n$ and $i_n$ are correlated.
  • Ignore that for now

• If $R_s$ is large:
  • Design amplifier with low $i_n$ (MOS)
• If $R_s$ is low:
  • Design amplifier with low $v_n$ (BJT)
• For a given $R_s$, there is an optimal $v_n/i_n$ ratio
  • Alternatively, for a given amp, there is an optimal $R_s$
Total Output Noise

\[ v_n^2 = (\frac{v_n^2}{R_u} + \frac{v_n^2}{R_f})^2 + \left(\frac{R_u}{R_u + R_f}\right)^2 \frac{v_n^2}{R_u} + v_n^2 \]

New Equivalent Generator

\[ v_{eq}^2 = v_n^2 + \frac{v_n^2}{R_u} \]

- With known \( R_u \), total noise can be lumped into one \( v_{eq} \)

Optimum Source Impedance

- Can use this to optimize source impedance for minimum added noise from two-port (noise figure):

\[ R_s = \frac{v_n^2}{4kT\Delta f} \quad G_s = \frac{i_n^2}{4kT\Delta f} \]

\[ R_{opt} = \sqrt{\frac{R_u}{G_n}} = \sqrt{\frac{v_n^2}{i_n^2}} \]

Correlated Noise Sources

- Partition \( i_n \) into two components:
  - Correlated ("parallel") to \( v_n \)
  - Uncorrelated ("perpendicular") to \( v_n \)

Correlated Noise Sources (cont.)

Finding \( Y_c \):

\[
\begin{align*}
\bar{v}_c^2 &= \bar{v}_n^2 + Z_c^2 \frac{\bar{v}_n^2}{Z_c^2} \\
&= \bar{v}_n^2 + Z_c^2 (v_c + i_c)^2 \\
&= \bar{v}_n^2 + Z_c^2 \bar{v}_c^2 + Z_c^2 \bar{i}_c^2 \\
&= \bar{v}_n^2 \left( 1 + Z_c^2 Y_c^2 \right) + Z_c^2 \bar{i}_c^2
\end{align*}
\]

\[ Y^2 = \frac{\bar{v}_c^2}{\bar{v}_n^2} \]

\[ Y_c^2 = \frac{\beta c^2}{\alpha c^2 \bar{v}_c^2 + \alpha c^2 \bar{i}_c^2} \]

Equivalent Noise Voltage (cor)

- Since the above expression is the sum of two uncorrelated noise voltages, we have

\[
\bar{v}_{eq}^2 = \bar{v}_n^2 + Y_c Z_s \bar{v}_n^2 + |Z_s| \bar{v}_n^2
\]

- Now we can continue as before to find

\[ B_{opt} = B_n = -B_c \]

\[ G_{opt} = G_n = \sqrt{\frac{G_n}{R_n} + G_c^2} \]