Why Modeling?

- Analog circuits more sensitive to detailed transistor behavior
  - Precise currents, voltages, etc. matter
  - Digital circuits have much larger “margin of error”

- Models allow us to reason about circuits
  - Provide window into the physical device and process
  - “Experiments” with SPICE much easier to do
Levels of Abstraction

- Best abstraction depends on questions you want to answer
  - Digital functionality:
    - MOSFET is a switch
  - Digital performance:
    - MOSFET is a current source and a switch
  - Analog characteristics:
    - MOSFET described by BSIM with 100’s of parameters?
    - MOSFET described by measurement results?

Detour: Device Corners

- Run-to-run parameter variations:
  - E.g. implant doses, layer thickness, dimensions
  - Affect $V_{TH}$, $\mu$, $C_{ox}$, $R_{on}$, ...
  - How model in SPICE?

- Nominal / slow / fast parameters (tt, ss, ff)
  - E.g. fast: low $V_{TH}$, high $\mu$, high $C_{ox}$, low $R_{on}$
  - Combine with supply & temperature extremes
  - Pessimistic but numerically tractable
  - $\rightarrow$ improves chances for working Silicon
Corner example: $V_{\text{TH}}$

- Corners just shift $V_{\text{th}}$
  - Probably not real
  - (PMOS doesn’t look real anyways)
- Variations probably bigger than reality too
  - Fab wants you to buy everything they make

Why not Square Law?

- Square law model most widely known:
  \[ I_{\text{D,sat}} = \frac{1}{2} \cdot \mu_n \cdot C_{\text{ox}} \cdot \frac{W}{L} \cdot (V_{GS} - V_{\text{th}})^2 \]
- But, totally inadequate for “short-channel” behavior
- Also doesn’t capture moderate inversion
  - (i.e., in between sub-threshold and strong inversion)
Square Law Model Assumptions

- Charge density determined only by vertical field
- Drift velocity set only by lateral field
- Neglect diffusion currents ("magic" $V_{th}$)
- Constant mobility
- And many more…

A Real Transistor

- **Gate Electrode**
  - Gate Depletion
  - Quantum Effect
- **Ultra-thin Gate Dielectric**
  - Direct Tunneling Current
  - Quantum Effects
- **S/D Engineering**
  - S/D resistances
  - S/D leakage
- **Retrograde Doping**
  - Body effect
- **Short Channel Effects**
  - Velocity Saturation and Overshoot
  - Source-end Velocity Limit
- **Pocket Implant**
  - Reverse short channel effect
  - Slower output resistance scaling with $L$
Now What?

- Rely purely on simulator to tell us how devices behave?
  - Models not always based on real measurements
  - Model extraction is hard
  - Models inherently compromise accuracy for speed

- Need to know about important effects
  - So that know what to look for
  - Model might be wrong, or doesn’t automatically include some effects
    - E.g., gate leakage

Output Resistance: CLM

- “Channel Length Modulation”
  - Depletion region varies with $V_{ds}$
  - Changes effective channel length

- If perturbation is small:
  \[
  I \propto \frac{1}{L - \delta L(V_{ds})} \approx \frac{1}{L} \left( 1 - \frac{\delta L(V_{ds})}{L} \right) \rightarrow \frac{I_{ds}}{I_{ds0}} = (1 + \lambda V_{ds})
  \]
Output Resistance: DIBL

• “Drain Induced Barrier Lowering”

• Drain controls the channel too
  • Charge gets imaged – lowers effective $V_{th}$
  • Model with $V_{th} = V_{th0} - \eta V_{DS}$

Output Resistance: SCBE

• “Substrate Current Body Effect”

• At high electric fields, get “hot” electrons
  • Have enough energy to knock electrons off Si lattice (impact ionization)

• Extra $e^{-}$ - $h^{+}$ pairs – extra (substrate) current
  • Models usually empirical

\[ I_{sub} = \frac{A_i}{B_i} I_{ds}(V_{ds} - V_{dsat})_{exp} \left( - \frac{B_i l}{V_{ds} - V_{dsat}} \right) \]
Output Resistance Mechanisms

- All effects active simultaneously
- CLM at relatively low fields
- DIBL dominates for high fields
- SCBE at very high fields

Velocity Saturation

- Drift velocity initially increases linearly with field
- Eventually carriers hit a “speed limit”
- In the limit, $I_D \propto (V_{GS}-V_{th})$
Vertical Field Mobility Reduction

- Mobility actually depends on gate field
  - “Hard to run when there is wind blowing you sideways (into a wall)”

- More technical explanation:
  - E-field pushes carriers close to the surface
  - Enhanced scattering lowers mobility

\[
\mu = \frac{\mu_0}{1 + \theta(V_{GS} - V_T) + \theta_B V_{SB}}
\]

Halo Doping

Source: R. Dutton and C.-H. Choi
Reverse Short-Channel Effect

Sub-Threshold Region

- Current doesn’t really go to 0 at $V_{GS} = V_{th}$
- Lateral BJT:
Weak Inversion Channel Potential

- “Base” controlled through capacitive divider
  \[ \delta V_{ch} \approx \frac{C_{ox}}{C_{dep} + C_{ox}} \delta V_g = \frac{\delta V_g}{n} \]

- Non-ideality factor of channel control \( n > 1 \):
  \[ n = 1 + \frac{C_{dep}}{C_{ox}} = 1 + \frac{\varepsilon_{dep}}{\varepsilon_{ox}} \]

- \( n \) varies somewhat with bias – const. approx. usually OK

Weak Inversion Current

- Current set by diffusion – borrow BJT equation:
  \[ I_{ds} = \frac{W}{L} I_{ds,0} e^{\frac{q(V_{gs}-V_{T})}{n k T}} \left(1 - e^{-\frac{q V_{ds}}{k T}}\right) \]
Operating in Weak Inversion

- Usually considered “slow”:
  - “large” $C_{GS}$ for “little” current drive (see later)

- But, weak (or moderate) inversion becoming more common:
  - Low power
  - Submicron L means “high speed” even in weak inversion

- Not well modeled, matching poor:
  - $V_{TH}$ mismatch amplified exponentially
  - Avoid in mirrors

Moderate Inversion

- Moderate inversion: both drift and diffusion contribute to the current.

- Closed form equations for this region don’t really exist.
Patching Models?

- Have “good” models for weak inversion and strong inversion.
  - Why not just interpolate in between?

- Example (EKV):

\[ I_{DS} = \frac{W}{L} \mu C_{ox}(2n) \left( \frac{kT}{q} \right)^2 \left( \ln \left( \frac{1}{1 - e^{\frac{v_{DS} - v_{th}}{2} - \frac{n}{2} V_{DD}}} \right) \right)^2 \left( \ln \left( \frac{1}{1 - e^{\frac{v_{DS} - v_{th}}{2} - \frac{n}{2} V_{DD}}} \right) \right) \]

BSIM

- *Berkeley Short-channel IGFET Model (BSIM)*
  - Industry standard model for modern devices
  - BSIM3v3 is model for this course

- Typically 40-100+ parameters
  - Advanced software and expertise needed to perform extraction
BSIM “Hand Calculation” Model

- Requires many, many, many… assumptions

- Vertical mobility degradation:
  Define: \( u_d = \frac{UA}{I_{ox}} \) mobility degradation coefficient

\[ u_d \approx 0.5V^{-1} \text{ for } t_{ox} = 10\text{nm} \]

- Velocity saturation:
  Define: \( E_c = \frac{2V_{sat}}{U/0} \) critical \( E \)-field for velocity saturation

\[ E_c \approx 2 \times 10^4 \text{V/cm} \text{ (typical value)} \]

Strong Inversion Current

\[ V_{Dsat} = \left( V_G - V_T \right) \left[ \frac{1}{1+u_d(V_G - V_T)} \right] \]

\[ I_{Dsat} = \mu_0 C_{ox} \frac{W}{L} \left[ \frac{1}{1+u_d(V_G - V_T)} \right] \]

\[ = I_{Dsat(long)} \left[ \frac{1}{1+u_d(V_G - V_T)+\left( \frac{V_G}{E_cL} \right)} \right] \]

\[ = I_{Dsat(long)} \left[ \frac{1}{1+u_d(V_G - V_T)+\left( \frac{V_G}{E_cL} \right)} \right] \]
Equations of Derivatives

\[
G_{\text{out}} = \frac{I_{\text{Dsat}}}{V_G-V_T} \left[ 1 + \frac{I_{\text{Dsat}}}{I_{\text{Dsat}\text{(long)}}} \right] = \frac{I_{\text{Dsat}}}{V_G-V_T} \left[ 1 + \frac{1}{u_d + \frac{1}{E_c L} (V_G-V_T)} \right]
\]

\[
r_{\text{out}} = \frac{2(V_D-V_{\text{Dsat}}) + [1 + u_d (V_G-V_T)](V_G-V_T)}{\mu_0 C_{ox} W P_{\text{CLM}}} \left[ 1 + u_d (V_G-V_T) \right] L^2
\]

\[
= \frac{(V_D-V_{\text{Dsat}}) + [1 + u_d (V_G-V_T)](V_G-V_T) L}{I_{\text{Dsat}\text{(long)}} P_{\text{CLM}}} \left[ 1 + u_d (V_G-V_T) \right]
\]

with \( l = \sqrt{3u_d x_f} \)

- Required parameters \( W, L, TOX, U_0, UA, VSAT, VTH0, PCLM, XJ \)

Fitting Results

Comparison between full and simplified model

Parameter detail: TSMC 0.18\(\mu\)m process

\( t_{ox}: 4.1\text{nm}, W=10\mu\text{m}, V_{TH0}=0.39\text{V} \)
Weakness of Model First Derivatives

![Graphs showing model comparison]

“Hand Model” Conclusion

- Even “simple” model is not convenient
  - $r_0$ is key for gain, but really hard to model
  - Missing important regions such as moderate inversion
- Hand models really best to build intuition
- But for design (i.e., how to choose $W$, $L$, etc.):
  - Will learn how to use the simulator as a “calculator”