Noise Variance in a Real Circuit: Sample and Hold

- Noise on the capacitor:
  \[ V_c(f) = 4kTf \frac{1}{1+sRC} \]
  \[ \Rightarrow V_d = \sqrt{V_c(f)} \frac{f}{kT} = \frac{kT}{C} \]
- So effective bandwidth is:
  \[ \Delta f = \frac{\pi f}{4RC} \]
  \[ \Rightarrow \Delta f \approx \frac{f}{2} \]

Sampled Signal Signal-To-Noise Ratio

- SNR:
  \[ SNR = \frac{P_{sig}}{P_{noise}} \]
- Signal Power (sinusoidal source):
  \[ P_{sig} = \frac{1}{2} V_{peak}^2 \]
- Noise Power (assuming thermal noise dominates):
  \[ P_{noise} = \frac{kT}{C} n_f \]
- So:
  \[ SNR = \frac{\frac{1}{2} CV_{peak}^2}{n_f kT} \]
  \[ SNR \uparrow +6dB \]
  \[ C \uparrow \times 4 \]

Energy-Based Analysis

dB versus Bits

- Quantization "noise"
  - Quantizer step size: \( \Delta \)
  - Box-car pdf variance: \( S_0 = \frac{S}{12} \)
  - SNR of N-Bit sinusoidal signal:
    \[ P_{sig} = \frac{1}{2} \left( \frac{2 \pi \Delta}{2} \right)^2 \]
    \[ SNR = \frac{P_{sig}}{S_0} = 1.5 \times 2^{1.8} \]
    \[ 6.02 \text{ dB per Bit} = \left[ \log_{10} \left( 0.001 + 6.02 N \right) \right] \text{ dB} \]
SNR versus Power

- 1 Bit $\rightarrow$ 6dB $\rightarrow$ 4x SNR
- 4x SNR $\rightarrow$ 4x C
- Circuit bandwidth $\rightarrow$ $\sim g_m/C$ $\rightarrow$ 4x $g_m$
- Keeping $V^*$ constant $\rightarrow$ 4x $I_D$, 4x W

- Thermal noise limited circuit:
  - Each bit QUADRUPLES power!
- Overdesign is expensive
  - Better do the analysis right!
  - Need to know how to get analytical expressions for more general circuits

Important Integrals

\[
\int_0^\infty \frac{1}{1 + \frac{s}{\alpha Q}} \, ds = \frac{\alpha Q}{\sqrt{\alpha^2 + 1}}
\]

\[
\int_0^\infty \frac{x}{\alpha Q} \left( 1 + \frac{s}{\alpha Q} \right)^{\frac{1}{2}} \, ds = \frac{\alpha Q}{\sqrt{\alpha^2 + 1}}
\]

\[
\int_0^\infty \frac{1}{\alpha Q} \left( 1 + \frac{s}{\alpha Q} \right)^{\frac{1}{2}} \, ds = \frac{\alpha Q}{\sqrt{\alpha^2 + 1}}
\]

Two-Stage Amplifier

Input Equivalent Noise

Equivalent Noise Generators

CS Amplifier

\[
\v_i(t) = \frac{1}{R_k} \left( \frac{2}{R} \left( \frac{R}{R_k + R, C} \right) + \frac{R}{R_k + R, C} \right) \delta(t)
\]

\[
\v_o(t) = \frac{1}{R_k} \left( \frac{2}{R} \left( \frac{R}{R_k + R, C} \right) + \frac{R}{R_k + R, C} \right) \delta(t)
\]

\[
= \frac{2}{R_k} \left( \frac{R}{R_k + R, C} \right) + \frac{R}{R_k + R, C}
\]

\[
= \frac{2 R}{R_k + R, C} + \frac{R}{R_k + R, C}
\]

\[
= \frac{3 R}{R_k + R, C}
\]

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\ell_o(t) = \frac{1}{R_k} \left( \frac{2}{R} \left( \frac{R}{R_k + R, C} \right) + \frac{R}{R_k + R, C} \right) \delta(t)
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\]

Two-Port Model for noisy two-port:
- Noiseless two-port
  - Plus equivalent input noise sources
- In general, $\ell_i$ and $\ell_o$ are correlated.
  - Ignore that for now
Finding the Equivalent Generators

- Find $v_n$ and $i_n$ by opening and shorting the input
  - Shorted input:
    - Output noise due only to $v_n$
  - Open input:
    - Output noise due only to $i_n$

Optimum Source Impedance

- Can use this to optimize source impedance for minimum added noise from two-port (noise figure):
  $$R_s = \frac{v_n^2}{4kTf} \quad G_s = \frac{i_n^2}{4kTf}$$
  $$R_{opt} = \sqrt{\frac{R_s}{G_s}} = \sqrt{\frac{v_n^2}{i_n^2}}$$

Role of Source Resistance

- If $R_s$ is large:
  - Design amplifier with low $i_n$ (MOS)
- If $R_s$ is low:
  - Design amplifier with low $v_n$ (BJT)
- For a given $R_s$, there is an optimal $v_n/i_n$ ratio
  - Alternatively, for a given amp, there is an optimal $R_s$

Correlated Noise Sources

- Partition $i_n$ into two components:
  - Correlated (“parallel”) to $v_n$
  - Uncorrelated (“perpendicular”) to $v_n$
- Can use this to re-derive optimum source $Z$

Total Output Noise

- $\Delta_i = (\frac{v_n^2}{2} + \frac{i_n^2}{2}) + \left( \frac{R_{in}}{R_{in} + R_s} \right)^2 \left( \frac{R_{in}}{R_{in} + R_s} \right)^2 \frac{v^2}{2} + \left( \frac{R_{in}}{R_{in} + R_s} \right)^2 \frac{i^2}{2}$

Loose Ends: Sampled Noise Spectrum

- What if RC doesn’t completely settle every cycle?
  - Noise between samples correlated $\rightarrow$ spectrum not white
  - If $Tr > 3$, correlation small
  - Sampled spectrum white
  - In practice usually the case
Loose Ends: Periodic Noise Analysis

SpectreRF PNOISE: check
noisetype=timedomain
noisetimepoint=[...]
as alternative to ZOH,
noisaskipcount=large
might speed up things in this case.